EXAMINATION OF THE $b$ COEFFICIENT IN GaP CRYSTAL UTILIZING SINGLE AND DOUBLE PHOTON ABSORPTION

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EXAMINATION OF THE $\beta$ COEFFICIENT IN GaP CRYSTAL UTILIZING SINGLE AND DOUBLE PHOTON ABSORPTION

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Abstract

A laser of 0.800 μm at 100 fs is utilized in obtaining a β coefficient in a GaP crystal associated with double photon absorption. Data is collected for the incident power of the laser for both single and double photon absorption, and it is plotted against the signal generated in the photodiode containing the GaP crystal. The slope of such a plot is employed in calculating β with equations derived in the theoretical model that is unique to the present work. The data for single photon absorption is utilized in calculating the thickness of the GaP crystal, while the data for double photon absorption is directly utilized in the calculation of β coefficient. The present work yields a numerical value for the absorption coefficient for double photon absorption, β.
The Theoretical Model

1.1 Introduction

A wide variety of phenomena is exhibited by the interaction of light and matter. As light enters an object, the light can scatter, reflect or refract. Sometimes the energy of the light transforms into heat or causes a chemical reaction. Other times, light can transmit through the media (Monk, 1963). The dominant phenomenon may depend on the type of material or the energy of the photons within the light beam. Our discussion will center on one of these types of phenomenon: absorption of light by matter. Specifically, this thesis focuses on the absorption of near infrared (IR) photons from a laser by Gallium Phosphide (GaP) material inside a photodiode. Within our GaP material, rare absorption process occurs called two-photon absorption alongside the common single photon absorption. This uncommon process will be the topic of this thesis.

Two-photon absorption (2PA) is a mechanism that is a subcategory of a much larger topic within nonlinear optics known as multi-photon absorption (MPA). A transition occurs in an MPA process that takes an atom from a lower energy level to a higher one by simultaneously absorbing multiple photons. It is a mechanism that is used to explain the effects of numerous photons interacting with matter in an instantaneous moment of time. If the number of photons being absorbed is three or four, then the mechanism is a 3PA or 4PA process. If the number of photons absorbed is two, then the mechanism is a 2PA or two-photon absorption process (He, 2015). 2PA process occurs when an electron within an atom simultaneously absorbs two photons, causing a transition to occur from the ground state to an excited state in an instant. This thesis uses the same apparatus to measure both 1PA and 2PA processes. We did not observe 3PA or higher order absorption processes because we lacked
Two-photon absorption (2PA) is a mechanism that is a subcategory of a much larger topic within nonlinear optics known as multi-photon absorption (MPA). A transition occurs in an MPA process that takes an atom from a lower energy level to a higher one by simultaneously absorbing multiple photons. It is a mechanism that is used to explain the effects of numerous photons interacting with matter in an instantaneous moment of time. If the number of photons being absorbed is three or four, then the mechanism is a 3PA or 4PA process. If the number of photons absorbed is two, then the mechanism is a 2PA or two-photon absorption process (He, 2015).

The 2PA process occurs when an electron within an atom simultaneously absorbs two photons, causing a transition to occur from the ground state to an excited state in an instant. This thesis uses the same apparatus to measure both 1PA and 2PA processes. We did not observe 3PA or higher order absorption processes because we lacked sufficiently high laser intensity.

The approach this work takes is to focus a laser onto a photodiode that contains a GaP crystal. Some of the laser photons are absorbed by the GaP crystal via 1PA and 2PA processes. Each absorption event is presumed to lead to the creation of a conduction electron which results in a measured current output. GaP crystal has a band gap of 2.26 eV, and this is the amount of energy required to free an outer-shell electron from its bind to the nucleus. Once a photon or photons are absorbed that satisfy the band gap, an electron in a semi-conductor becomes a conduction electron due to the photon absorption, generating a current inside the photodiode. This signal from the photodiode representing the current is recorded and stored where it will be used to generate plots of GaP Signal vs. Incident Power of the laser. The incident intensity of the laser is controlled by a half-waveplate and a linear polarizer through which the beam passes on its way to the photodiode. This waveplate is mounted on top of a stepper motor that allows the waveplate to rotate in a circle. The intensity of the light beam follows Malus's law such that the intensity of the laser, $I$, varies as the waveplate angle, $\theta$, is rotated due to the relationship of $I = I_0 \cos^2 \theta$ where the intensity of the laser is dependent on the angle of rotation. This variable incident power is recorded and stored. This data on the laser’s power is later plotted against the data from the photodiode. The GaP photodiode current is proportional to the number of electrons participating in the absorption of the laser’s photons. Plotting GaP Signal vs. Incident Power allows for the 1PA and 2PA behaviors to be explored. In particular the power-law dependence is used to identify 1PA and 2PA, and a more detailed analysis allows the associated absorption coefficient, $\beta$, to be determined.

The absorption coefficient, $\beta$, was determined through a theoretical model whose structure will be discussed in the subsequent sections. The theoretical model depends on utilizing the slopes which are generated through plotting of data of the photodiode’s current against the laser’s incident power. The slopes of these graphs of GaP Signal vs Incident Power for the single photon absorption are used in determining the thickness of the GaP crystal inside the photodiode. The thickness of the GaP crystal is a necessary piece of information that needs to be known in order to calculate $\beta$. Of course, $\beta$ is the absorption coefficient for double photon absorption whose
be known in order to calculate $\beta$. Of course, $\beta$ is the absorption coefficient for double photon absorption whose numerical value we seek to determine in this thesis. The thickness of the GaP crystal obtained through the slopes of plots from single photon absorption is used alongside the slopes generated by the data of double photon absorption in order to calculate $\beta$. The slopes generated by double photon absorption also stem from plots of GaP Signal vs Incident Power and are necessary in the calculation of $\beta$. We will begin by presenting the theoretical model on which the derived equations are based on which allows the calculation of $\beta$. From there, the obtained data with its respective slopes will be analyzed and the derived equations will be used in acquiring a numerical value for $\beta$. Through the comparison with prior publications, the validity of $\beta$ for the present work can be gauged.

1.2 Single Photon Absorption

In this section, equation for the single-photon absorption (1PA) occurring in the GaP material is derived. The ending result of the derivation is Equation 1.1 which summarizes the phenomena of 1PA processes of our experiment. The following subsections will present how Equation 1.1 was derived since it is the equation within our theoretical model that allows 1PA process to be confirmed in our data using scaling. Equation 1.1 is also utilized in the calculation of the thickness of the GaP crystal, $z$, which is necessary for calculating $\beta$ for our present work. The “1PA equation” for our theoretical model is

$$i = \left( \frac{\lambda}{hc} \right) P_o \alpha z . \tag{1.1}$$

This equation shows a relationship between the incident power of the laser, $P_o$, and the current produced by the GaP material, $i$. The $\lambda$ is the wavelength of the photons which is 800 nm of our laser, and $hc$ is the Planck’s constant and the speed of light. The constant, $\alpha$, is the absorption coefficient for single photon absorption whose value is 0.54 /cm for a GaP material as published by Chong, Watson, and Festy. Keeping in mind, $\alpha$ is different from $\beta$ coefficient since $\beta$ is associated with double photon absorption (2PA) in which $\alpha$ will be utilized in the calculations of the experimental $\beta$.

Single photon absorption occurs when the energy of the laser is greater than the band gap of the GaP crystal found inside the photodiode. When this energy gap is overcome through absorption of a photon, the electron
that is bound to the atom can become a conduction electron. In 1PA process, it is assumed that it takes only one photon in order to produce one conduction electron. This loosely bound electron inside a semiconductor such as a GaP crystal can contribute to the current flowing inside the material. The current is the signal generated by the photodiode and received by the voltage amplifier. This is also the same signal that will be plotted against the incident power of the laser, allowing a slope of the graph to be determined using Matlab. The slope of this graph is utilized in calculating the thickness of the GaP crystal, $z$, using Equation 1.1. In Equation 1.1, dividing $i$ with $P_o$ gives a slope of the graph of GaP Signal vs. Incident Power of the Laser. Placing this experimental slope into Equation 1.1, the value for $z$ can be calculated since all other variables are known. The thickness of the GaP crystal, $z$, must be known in order to calculate $\beta$ coefficient for 2PA. Without it, $\beta$ cannot be experimentally determined. Therefore, the data collected for 1PA process is a necessary step for determining $z$ and subsequently $\beta$.

The approach taken in the derivation of Equation 1.1 was to subtract the final number of photons in the laser beam from the initial number. The difference in the number of photons corresponds to the amount of photons being absorbed. Logic would dictate that the difference would be equal to the amount of photons lost due to propagation through the material and the equipment; hence, absorption.

The derivation of Equation 1.1 began with discerning the number of photons that emerges out of the laser. This incident number of photons is constructed by dividing the initial power of the laser by the energy of one photon. When the total energy is divided by a photon’s energy, a number of photons are given. An equation is derived for the initial number of photons using this reasoning. With the incident number of photons now known, the last piece to be derived is the final number of photons in the light beam as it penetrates the crystal. As the beam travels further into the material, the number of photons decreases exponentially. This exponential decay of photons is given by Beer’s law which is discussed in Section 1.2.2. Using Beer’s law to represent the decrease of the amount of photons, an equation for the final number of photons is obtained. The number of photons that is absorbed by the GaP material during 1PA is found when the initial and the final number of laser’s photons is subtracted, leading to equation of 1.1. The upcoming Sections of 1.2.1 through 1.2.4 discuss
the derivation of Equation 1.1 or the “1PA equation.”

1.2.1 Photons Incident on Material

Single Photon Absorption (1PA) is employed within the theoretical model for the sole purpose of calculating the thickness of the GaP material within the photodiode, which is denoted by the symbol $z$. From the earlier publications, the experimental value for the linear absorption coefficient, $\alpha$, is known to be around 0.54/cm (Yee and Chau, 1974). As previously stated, $\alpha$ is the absorption coefficient for single photon absorption unique to GaP crystal; whereas, $\beta$ is the absorption coefficient for double photon absorption. All of this information is used to derive an equation that represents the 1PA phenomenon given by Equation 1.1. Considering that one electron is generated for each photon absorbed in 1PA, it made logical sense to structure an equation that represents a linear relationship between the incident laser power, $P_o$, and the photocurrent inside the GaP crystal, $i$. This process of derivation begins by contemplating how the photons from the laser interact with the material.

There are a certain number of photons per second that are incident on the GaP material within the photodiode. That number, expressed by $N_o$, is obtained by dividing the total power of the laser by the energy of a single photon within the beam: $E_{\text{photon}} = \frac{hc}{\lambda}$. The incident wavelength of light, $\lambda$, is approximately 800 nanometers, and $hc$ is the Planck constant multiplied by the speed of light in a vacuum. Since the intensity of the laser is known, the total power is attained when the intensity is multiplied with the area of the beam:

$$\pi (w_o)^2 I_o = P_{\text{laser}}.$$  

The area of the beam is circular in nature and is given by $\pi w_o^2$ where $w_o$ is the beam spot size. With all of this combined, we got an equation for the number of photons per second incident of the GaP material,

$$N_o = \left(\frac{\lambda}{hc}\right) \pi w_o^2 I_o.$$  \hspace{1cm} (1.2)

Seeing that the experiment revolves around an absorption coefficient, the interest is placed on the photons being absorbed and not on the incident photons. Equation 1.2 gives a value for the number of photons that are
directly emitted out of the laser. To derive the number of photons absorbed by the GaP material, the number of incident photons is subtracted by the exponential associated with attenuation, obtaining the following representation \(1 - e^{-\alpha z}\). As light passes through an absorbing material, the strength of the light's energy decreases as it penetrates deeper and deeper into the material (Williamson, 1983). This occurrence is described by the decaying exponential function which is accomplished through the use of Beer's Law.

1.2.2 Beer's Law: Impact on Intensity due to the Propagation through the GaP Crystal

This type of exponential decay appears in situations where the decrease of the quantity in a given distance is proportional to its initial value (Williamson, 1983). This means that the rate of change of light's intensity is proportional to the intensity itself, and this statement for IPA is represented as

\[
\frac{dI}{dz} = -\alpha I.
\] (1.3)

In the above equation, \(\alpha\) is the linear absorption coefficient, \(z\) is the depth of the light's travel into the material, and \(I\) is the intensity of the light. When this differential equation is solved, Beer's law is derived. So, let us derive Beer's law by the use of separation of variables. The intensity is separated from the variable of depth,

\[
dI = -\alpha I \, dz.
\]

The intensity is moved to the left-hand side of the equation, while the depth of penetration is moved to the right-hand side of the equation. This gives us

\[
\frac{dI}{I} = -\alpha \, dz.
\]

Isolation of the variables allows for a successful integration to be performed. After the variables are separated, integrating both sides of the equation provides a solution to the differential equation where \(C\) is the constant produced from integration.

\[
\int \frac{dI}{I} = \int -\alpha \, dz.
\]

We pull the coefficient outside of the integral since it is not a variable. We obtain
\[
\int \frac{dI}{I} = -\alpha \int dz.
\]

The integration provides a natural log in the equation,

\[
\ln(I) = -\alpha z + C.
\]

Next, some basic algebra steps are followed in order to eliminate the presence of the natural log. When both sides of the equation are raised by an exponential, a relationship is obtained between the intensity of the light and the depth of travel,

\[
e^{\ln(I)} = e^{-\alpha z + C}.
\]

The exponential eliminates the need for the natural log. This leaves us with

\[
I = e^{-\alpha z} e^C
\]

and

\[
I = (C) e^{-\alpha z}.
\]

The value for the constant is obtained since it is known that the intensity of the light at the entrance of the material, or when \(z\) is equal to zero, is given as \(I_o\). If these boundary conditions are inserted into the derivation, the final conclusion is reached,

\[
I_o = (C) e^{-\alpha(0)}
\]

and

\[
I_o = C.
\]

Beer’s Law is finally derived when the boundary conditions are accounted for. The final form of the Beer’s law is given as

\[
I = I_o e^{-\alpha z}.
\]

Beer's law gives the mathematical representation of the interaction between the material and the light, giving the intensity of the light, \(I\), at a particular thickness of the material, \(z\). After the beam of light is incident upon the material, each layer of thickness absorbs the same relative amount of the light as it travels through the medium (Monk, 1963). The intensity of light is decreased exponentially the further it travels into the GaP material. Since the number of photons is directly proportional to the intensity, the number of photons at a given depth of the material is given by
\[ N = N_0 e^{-ax}. \] (1.5)

1.2.3 Photons Absorbed by the GaP Crystal

The number of photons absorbed by the material is calculated by using Beer's law and the number of incident photons given in equation 1.2. When the initial and the final number of photons are subtracted from each other, the answer should yield the number of photons absorbed. Naturally, the final number of photons is dependent on the depth of the light's propagation through the material and is reliant on Beer's law. For this portion of our derivation, we obtain the following when equation 1.2 was subtracted from equation 1.5

\[ N_{\text{absorbed}} = N_0 - N_0 e^{-ax}. \]

When the \( N_0 \) is factored out, the equation is

\[ N_{\text{absorbed}} = N_0(1 - e^{-ax}). \]

We make a substitution for \( N_0 \) using equation 1.2,

\[ N_{\text{absorbed}} = \left( \frac{\lambda}{\hbar c} \right) \pi w_0^2 I_o (1 - e^{-ax}). \]

Since the intensity of the light is just power per area, a substitution can be made into the derivation which creates an equation dependent on the average power of the laser, \( P_o = \pi w_0^2 I_o \). The equation becomes

\[ N_{\text{absorbed}} = \left( \frac{\lambda}{\hbar c} \right) P_o (1 - e^{-ax}). \]

One electron is created for every photon absorbed by the GaP material in 1PA process. Knowing this, we can state that the number of photons absorbed is equivalent to the numbers of electrons generated within that time period. Therefore, we will make one final substitution between number of photons absorbed and the current of electrons, \( i = N_{\text{absorbed}} \). We finally reach the final step,

\[ i = \left( \frac{\lambda}{\hbar c} \right) P_o (1 - e^{-ax}). \] (1.6)

Equation 1.6 presents a relationship between the incident laser power and the photocurrent, which are both experimental values that were measured in the laboratory. To be precise, the photocurrent was not measured directly. Instead, the output of a voltage amplifier was recorded in lieu of it. The signal voltage \( v \) produced by
the voltage amplifier is related to the photocurrent $i$ by a gain factor. This gain factor, $G$, will be used in later sections to convert the equation into the appropriate units.

### 1.2.4 Simplification of the Derived Equations for the Single Photon Absorption

Even though the equation 1.6 is sufficient enough for the general usage, the equation does encounter complications if used with natural logarithm. Our scaling theory, which is discussed in later sections, does involve the use of natural logarithm on very small numbers. This, in turn, causes issues to come about since very small numbers are usually seen as zeros by calculators and computing software such as Matlab. Natural log of a zero is not a good element to have within a set of calculations. In order to avoid mathematical errors, our equation for 1PA is simplified through the use of the Taylor Series for exponentials,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.$$  

The Taylor series for exponentials will be substituted into the equation for 1PA that was previously derived. Since not all the terms of the expansion are vital, the series will be truncated at the second term. The results of the expansion are

$$i = \left(\frac{\lambda}{hc}\right)P_o\left(1 - \left(1 - (-\alpha z) + \frac{(-\alpha z)^2}{2!} + \frac{(-\alpha z)^3}{3!} + \ldots\right)\right)$$

and

$$i = \left(\frac{\lambda}{hc}\right)P_o\left(1 - 1 - (-\alpha z) - \frac{(-\alpha z)^2}{2!} - \frac{(-\alpha z)^3}{3!} + \ldots\right).$$

We truncate the series so that we get

$$i = \left(\frac{\lambda}{hc}\right)P_o(1 + \alpha z),$$

and once we simplify,

$$i = \left(\frac{\lambda}{hc}\right)P_o \alpha z.$$  

Equation 1.8 explains the 1PA process within the GaP material where $\lambda$ and $hc$ are known. $P_o$ and $i$ are the experimental values which are obtained through measurements and $\alpha$ is the linear absorption coefficient provided by Yee and Chau. From this information, $z$ is calculated and used to find the 2PA coefficient, $\beta$. 

Equation 1.8 allows us to calculate \( z \) and \( \beta \) without encountering mathematical errors associated with small numbers and natural logs.

### 1.3 Double Photon Absorption

Double Photon Absorption (2PA) occurs when the energy of the laser is greater than half of the band gap of the GaP crystal, \( E_{\text{laser}} > (1/2) E_{\text{BG}} \). When this energy gap is overcome through absorption of two photons, the electron that is bound to the atom can become a conduction electron. In 2PA process, it takes two photon in order to produce one conduction electron. The conduction electron inside a GaP crystal can contribute to the current flowing inside the material which produces a signal inside a photodiode. This signal will be plotted against the incident power of the laser, allowing a slope of the graph to be determined using Matlab. When \( i \) is divided by \( (P_o)^2 \), a slope of the graph of GaP Signal vs. Incident Power of the Laser is produced. When this experimental slope is inserted into a “2PA equation”, the value for experimental \( \beta \) is calculated. This procedure is very similar to the method behind 1PA and calculating a value for \( z \).

#### 1.3.1 What is 2PA?

Imagine a laser beam of frequency \( \nu \) passing through a nonlinear medium in which a 2PA process occurs, resulting in two photons being absorbed into the medium. If one is to witness such an event, the blueprint of that process looks like the illustration below. In Figure 1.1, the mechanism of such a process is shown as a "two-step" event with each event being marked by an absorption of a single photon (He, 2015).
Figure 1.1. Description of the mechanism of two-photon absorption. A single photon is absorbed which causes the atom to transition from the ground state to an intermediate state. From there, another photon is absorbed which causes the atom to transition from intermediate state to an excited state. This process occurs simultaneously.

Furthermore, between the ground state and the excited state of an atom, there is an intermediate state that separates the two step process as well. In the first step, a single photon is absorbed which causes the electron to transition from ground state into the intermediate state. While at the intermediate state, the electron will wait for the second photon's interaction. The second step involves absorbing the second photon which causes a transition from the intermediate state to the final excited state. The "two-step" procedure of the 2PA process happens instantaneously where both steps occur concurrently. Not only is this the blueprint behind the 2PA mechanism, but it is the general explanation behind multi-photon absorption found in nonlinear optics. This same blueprint can be applied to 3PA (or 4PA) process where three (or four photons) are simultaneously absorbed through a procedure involving two (or three) intermediate steps as shown in Figure 1.2. For even higher number of photons being simultaneously absorbed, the mechanism is still valid with each level of MPA involving more intermediate steps (He, 2015) A schematic representation of the processes 3PA and 4PA can be seen in Figure 1.2. The reverse process of MPA is also achievable where an atom goes from an excited state to the ground state. In this scenario, several photons are created and emitted simultaneously as the atom transitions from upper to lower energy level (Schubert, 1986).
Naturally, conservation of energy requires that the difference of energy between the two eigenstates be equal to the energy of photons; therefore, for the process of 2PA and 3PA, the difference should yield $E_f - E_i = 2\hbar\nu$ and $E_f - E_i = 3\hbar\nu$, respectively. Combining the energies of each photon at every intermediate step gives the correct depiction of the overall process at the instantaneous moment of its occurrence (He, 2015). Additionally, the energies of the photons do not have to be identical to each other. It is possible and allowable to have an MPA process that involves absorption of different frequencies of photons. The term *nondegenerate MPA* is designated to the process that involves a beam containing two or more frequencies which leads to the simultaneous absorption of different frequencies of photons. *Degenerate MPA* is, of course, reserved for defining those processes involving the same frequency photons (He, 2015).

### 1.3.2 History behind Double Photon Absorption

Two photon absorption was initially proposed by Goppert-Mayer in 1931 who utilized quantum mechanics to theoretically predict the transition between two eigenstates under the influence of 2PA process (He, 2015). This section covers some of the historical aspects of her work. For those not interested, this section can be skipped. For those interested, a brief discussion will be made regarding her dissertation which theoretically predicted two-photon absorption.

Applying second-order perturbation, Goppert-Mayer calculated the transition probability associated with two photons causing an atom to become excited after absorption (Goppert-Mayer, 1931). Generally speaking,
the probability of finding the state $\Psi$ in a measurement performed on the state $\phi$ is $|\langle \Psi | \phi \rangle|^2$ where $\langle \Psi | \phi \rangle$ is the probability amplitude for the transition (Dick, 2012). In her publication of 1931, Maria Goppert-Mayer calculated the transition probability of two-photon processes to be of the form (Schubert, 1986)

$$P_{i \rightarrow f}(t) = \left| \frac{1}{(\hbar t)^2} \sum_j \frac{\langle E_i | H | E_j \rangle \langle E_j | H | E_f \rangle}{E_f - E_j} G_2(t) \right|^2$$

The initial and final states of the process are given by $| E_i \rangle$ and $| E_f \rangle$ where as the intermediate state of the process is represented by $| E_j \rangle$. As can be observed from the equation, all of the intermediate levels represented by index $j$ are summed up and the representation for $G_2(t)$ which is the function that carries the time dependence is given as (Schubert, 1986)

$$G_2(t) = \frac{\hbar \langle E^n_{i+1} \rangle}{w_i} - \frac{\hbar \langle E^n_{i+1} \rangle}{w_j}$$

In this instance, $w_i$ and $w_j$ represent the transition frequencies between the final and initial state and between the final and intermediate state (Schubert, 1986). Utilizing slightly different notation and symbolism, Goppert-Mayer derived the probability amplitudes, $a_{ns}$, which are necessary for calculating probability transition given above. In equation (3) of her publication, she presents the first and second order probability amplitudes (Goppert-Mayer, 1931).

$$(a_{ns})^1 = \left( H_{n,s,n',s'} \right) \frac{1-e^{2\pi i n o + \nu_{s,s'}}}{\hbar (\nu_{n,n'} + \nu_{s,s'})}$$

$$(a_{ns})^2 = \sum s' \left( H_{n,s,n',s'} (H_{n,s',s'}) \frac{1-e^{2\pi i n, o + \nu_{s,s'}}}{\hbar (\nu_{n,n'} + \nu_{s,s'})} - \frac{1-e^{2\pi i n o + \nu_{s,s'}}}{\hbar (\nu_{n,n'} + \nu_{s,s'})} \right)$$

The above equations give the first and second order probability amplitudes of $(a_{ns})^1$ and $(a_{ns})^2$ that Goppert-Mayer calculated. Her notation is slightly different where she uses $H_{n,s,n',s'}$ as a representation of the matrix element connected to the perturbation Hamiltonian. Furthermore, she identifies $s$ as the eigenstate of one of the photons being absorbed and $n$ as the state associated with the other photon, so that $E_f - E_i = 2\hbar = \hbar (\nu_{n,n'} + \nu_{s,s'})$. The superscripts associated with $s$ and $n$ indicate the energy levels so that $n^o$ is the initial state, $n$ is the final state, and $n'$ is the intermediate state (Goppert-Mayer, 1931).
1.3.3 Impact of Intensity due to Propagation through the GaP Crystal

In order to obtain the number of photons absorbed by the GaP material, the rate at which the intensity of the beam changes as it propagates through the matter must be examined. Since the 2PA process triggers two photons to be absorbed simultaneously, the rate of change for intensity will no longer be the same as the rate for 1PA. The rate of change of the intensity of the beam of light is now proportional to the square of the intensity because two photons are absorbed in order for one conduction electron to be generated. This relationship is

\[ \frac{dI}{dz} = -\beta I^2. \]

The solution to this differential equation will provide a value for intensity of light that is dependent on \( z \), the thickness of the material or the depth of the penetration. As the light beam travels into the material, the number of photons in the beam will decrease. Through the method of separation of variables, this solution for the intensity is

\[ dI = -\beta I^2 \, dz. \]

The two variables are separated from each other in order to allow integration to be possible,

\[ \frac{dI}{I^2} = -\beta \, dz. \]

We have separated the variables from each other. Next, we integrate both sides of the equation in order to gain the desired relationship between the two variables of \( I \) and \( z \),

\[ \int \frac{dI}{I^2} = -\beta \int dz. \]

Basic manipulation is used to solve the integral,

\[ \frac{I^{-1}}{-1} + C = -\beta z \]

and

\[ \frac{1}{I^{-1}} + C = -\beta z. \]

We can apply the boundary conditions in order to solve for the constant, \( C \). At the surface of the GaP crystal, the intensity of the beam is \( I_0 \) at \( z = 0 \). These are the same initial conditions that were used in the derivation of 1PA and Beer’s Law. Applying these initial conditions to the differential equation, we obtain

\[ \frac{1}{I_0} + C = -\beta (0). \]
Solving the equation, we get the value for $C$ as

$$\frac{1}{I_0} = C.$$ 

We can substitute the constant back into the equation, and solve for $I$ in order to get the final intensity of the beam as it penetrates the medium,

$$\frac{-1}{I} + \frac{1}{I_0} = -\beta z.$$ 

Some basic algebra is used to clean up the equation. This gives us

$$\frac{-1}{I} \left(\frac{L_z}{I_0}\right) + \frac{1}{I_0} \left(\frac{I}{I_0}\right) = -\beta z$$

and

$$\frac{-I_0 + I}{I_0} = -\beta z.$$ 

We need to isolate the intensity variable from rest of the terms. We do the following to achieve that goal,

$$-I_0 + I = -\beta z I(I_0)$$

and

$$-I_0 = -\beta z I(I_0) - I.$$ 

We continue and solve for $I$ in our derivation,

$$-I_0 = I (-\beta z I_0 - 1)$$

and

$$\frac{I_0}{\beta z I_0 + 1} = I.$$ (1.9)

Equation 1.9 provides the relationship between the depth of penetration of the laser beam, $z$, and its intensity. This relationship only applies for the events associated with two-photon absorption.

### 1.3.4 Photons Absorbed by the GaP Crystal during Double Photon Absorption

The theoretical model for double photon absorption follows similar logic for the derivation as the single photon absorption. Initially, it is necessary to know the number of photons that are incident on the material. This information was calculated beforehand in section 1.2.1, and the results will merely be restated here. For the initial number of photons carried by the laser beam, we have Equation 1.2 rewritten below,
\[ N_0 = \left( \frac{\lambda}{\text{hc}} \right) \pi w_0^2 I_0. \]

Equation 1.9 is the intensity of the light beam at a depth \( z \) of the GaP material. This alteration of intensity, caused by penetration of the material by the light, is used to find the final number of photons. If this value is subtracted from the initial intensity, then information can be gained regarding how many photons are absorbed by the material during a 2PA process. Therefore, similar procedure is executed for finding the number of absorbed photons as in section 1.2.3 for a 1PA process. Utilizing Equation 1.9, the derivation for such information is

\[ N_{\text{abs}} = N_{\text{initial}} - N_{\text{final}}. \]

Substitution is made for \( N_{\text{initial}} \) and \( N_{\text{final}} \) so that we get

\[ N_{\text{abs}} = \left( \frac{\lambda}{\text{hc}} \right) \pi w_0^2 I_0 - \left( \frac{\lambda}{\text{hc}} \right) \pi w_0^2 \left( \frac{I_0}{\beta z L_{L+1}} \right). \]

If we pull out the constant, we get a more simplified equation,

\[ N_{\text{abs}} = \left( \frac{\lambda}{\text{hc}} \right) \pi w_0^2 I_0 \left( 1 - \frac{1}{\beta z L_{L+1}} \right). \]

The intensity is related to the power and the area as \( I = P_p / \pi w_0^2 \). The variable of \( P_p \) represents the peak power of the laser under which double photon absorption occurs. Peak power of the laser is utilized in the data collection of 2PA processes because intensity is greater under peak power. This makes it more likely for double photon absorption to occur when intensity is greater since 2PA demands two photons for absorption. As the intensity of the laser is increased, higher order absorptions such as 3PA and 4PA would be witnessed. Because of the intensity dependence, 1PA processes tends to dominate when the laser is in continuous-wave mode while 2PA dominates in pulsed mode of the laser.

This substitution is made directly into the derivation in order to produce an equation that is compatible with a \( I \) vs \( P_p, 2 \) graph necessary for finding the slope, meaning a GaP Signal vs Incident Power. The substitution for power is required at this point since our data is in terms of the power of the laser as stated in the introduction. We get

\[ N_{\text{abs}} = \left( \frac{\lambda}{\text{hc}} \right) \pi w_0^2 \left( \frac{P_p}{\pi w_0^2} \right) \left( 1 - \frac{1}{\beta z \left( \frac{r_e}{w_0^2} \right)^{L_{L+1}}} \right) \]

and
\[ N_{\text{abs}} = \left( \frac{\lambda}{2hc} \right) P_p \left( 1 - \frac{1}{\beta z \left( \frac{\rho_v}{\mu_w} \right) + 1} \right). \]

For the process of two-photon absorption, one electron is generated for every two photons absorbed. This association needs to be accounted for in the derivation which is why a factor of 2 is inserted in front of the energy of photon seen in Equation 1.10,

\[ N_{\text{abs}} = \left( \frac{\lambda}{2hc} \right) P_p \left( 1 - \frac{1}{\beta z \left( \frac{\rho_z}{\mu_w} \right) + 1} \right). \] (1.10)

Equation 1.10 is derived for two-photon absorption that gives the number of photons absorbed during the phenomenon. Further steps need to be taken in order to simplify the equation mathematically. The calculations would be less difficult if Equation 1.10 were simplified into a more compacted form. Since the natural log will be taken of Equation 1.10, it would be a lot easier if the equation can be simplified in a manner that makes it easier to analyze without generating errors associated with natural logs of small numbers. Therefore, an attempt will be made to simplify Equation 1.10.

### 1.3.5 Simplification of the Derived Equation of the Double Photon Absorption

Simplifications can be made to the derived Equation 1.10 for two-photon absorption in order to make it easier to manage mathematically. Basic algebra can be used to clean up the equation,

\[ N_{\text{abs}} = \left( \frac{\lambda}{2hc} \right) P_p \left( \frac{\beta z \left( \frac{\rho_v}{\mu_w} \right) + 1}{\beta z \left( \frac{\rho_v}{\mu_w} \right) + 1} \right) \]

and

\[ N_{\text{abs}} = \left( \frac{\lambda}{2hc} \right) P_p \left( \frac{\beta z \left( \frac{\rho_z}{\mu_w} \right) + 1}{\beta z \left( \frac{\rho_z}{\mu_w} \right) + 1} \right) \]

and

\[ N_{\text{abs}} = \left( \frac{\lambda}{2hc} \right) P_p \left( \frac{\beta z \left( \frac{\rho_z}{\mu_w} \right)}{\beta z \left( \frac{\rho_z}{\mu_w} \right) + 1} \right). \]

The algebraic manipulations permitted the equation to be placed in a format compatible with the use of Taylor series. The portion of the equation inside the large parenthesis can now be expanded around \( x \) equal to
The algebraic manipulations permitted the equation to be placed in a format compatible with the use of Taylor series. The portion of the equation inside the large parenthesis can now be expanded around $x = -\beta z$ and then truncated at the appropriate term of the series. In addition, we can substitute $N_{\text{abs}}$ with $i$ for the reason that the number of photons absorbed are connected to the electrons being released by the material. We get

$$i = \left(\frac{\lambda \beta z}{2hc}\right) P_p \beta z \left(\frac{P_p}{\pi w_o^2}\right) \left(\frac{1}{\beta z \left(\frac{P_p}{\pi w_o^2}\right) + 1}\right)$$

and

$$i = \left(\frac{\lambda \beta z}{2hc}\right) \left(\frac{P_p^2}{\pi w_o^2}\right) \left(\frac{1}{\beta z \left(\frac{P_p}{\pi w_o^2}\right) + 1}\right)$$

and

$$i = \left(\frac{\lambda \beta z}{2hc}\right) \left(\frac{P_p^2}{\pi w_o^2}\right) \left(\frac{1}{1 - \beta z \left(\frac{P_p}{\pi w_o^2}\right)}\right).$$

The Taylor series utilized in the expansion of the “2PA equation” is presented below. By setting $x$ equal to $-\beta z \frac{P_p}{\pi w_o^2}$, the Taylor series expands the “2PA equation,”

$$\frac{1}{1-x} = \sum x^n = 1 + x + x^2 + x^3 ... + ... x^n .$$

We make a substitution for $x$ so that we get

$$\frac{1}{1 - (-\beta z \frac{P_p}{\pi w_o^2})} = 1 + \left(-\beta z \frac{P_p}{\pi w_o^2}\right) + \left(-\beta z \frac{P_p}{\pi w_o^2}\right)^2 + \left(-\beta z \frac{P_p}{\pi w_o^2}\right)^3 ... + ... \left(-\beta z \frac{P_p}{\pi w_o^2}\right)^n$$

and

$$\frac{1}{1 - (-\beta z \frac{P_p}{\pi w_o^2})} = 1 - \beta z \frac{P_p}{\pi w_o^2} + \left(-\beta z \frac{P_p}{\pi w_o^2}\right)^2 + ... + ... .$$

The expansion is substituted into the “2PA equation,” and the results are truncated after the first term since the most significant contribution for the Taylor series rests on the first term. The remainder of the higher-order terms in the Taylor series are exploited only when greater approximations are necessary for the theoretical model. It is enough for the truncation to produce a first order $\beta$. For our model, the first term will suffice,

$$i = \left(\frac{\lambda \beta z}{2hc}\right) \left(\frac{P_p^2}{\pi w_o^2}\right) \left(1 - \beta z \frac{P_p}{\pi w_o^2}\right).$$

We terminate certain terms to get
\[ i = \left( \frac{\lambda \beta z}{2 \hbar c} \right) \left( \frac{P_p^2}{\pi w_o^2} \right) - \left( \frac{\lambda \beta z}{2 \hbar c} \right) \left( \frac{P_o^2}{\pi w_o^2} \right) \beta z \frac{P_p}{\pi w_o^2} \]

and

\[ i = \left( \frac{\lambda \beta z}{2 \hbar c} \right) \left( \frac{P_p^2}{\pi w_o^2} \right) . \]

The peak power, \( P_p \), is used when the laser is in pulsed mode to find \( \beta \) for two-photon absorption. The relationship between peak power and average power of the laser is given by \( P_p = P_o / (f \Delta t) \) where \( \Delta t \) is the pulse width of the laser at 100 fs and \( f \) is the repetition frequency at 100 MHz. The plots are graphs of \( i \) vs \( P_o \), making the substitution necessary. When this association is accounted for in the derivation, the following is determined,

\[ i = \left( \frac{\lambda \beta z}{2 \hbar c} \right) \left( \frac{P_o^2}{f \Delta t \pi w_o^2} \right) \]

and

\[ \frac{i}{P_o^2} = \left( \frac{\lambda \beta z}{2 \hbar c} \right) \left( \frac{1}{f \Delta t} \right)^2 \left( \frac{1}{\pi w_o^2} \right) . \] (1.11)

Keeping in mind that the left-hand side of the equation is the slope \( m \), the “2PA equation” given by Equation 1.11 is ready to be utilized in the calculations of \( \beta \) coefficient. Ideally, the experimental data should yield a value for \( \beta \) that is close to the values in previous publications of \( 1.7 \times 10^{-3} \) and \( 2 \times 10^{-4} \) cm/MW (Yee, 1974, Bechtel, 1976). Their publications will be used as a comparison for the experimental \( \beta \) produced by the present work even though their methodology was different than the present work. Both publications used optical methods to measure the value of \( \beta \) where an incident laser beam was propagated through a GaP crystal, and the output intensity was evaluated; whereas, the present work utilized electrical method of evaluating an experimental \( \beta \).

1.3.6 Scaling of Single and Double Photon Absorption

Preferentially, it would be both practical and constructive to verify that the experimental set up chosen is effective in measuring single and double photon absorption. The data needs to confirm that 1PA and 2PA process are in fact happening inside the photodiode, and this confirmation of the phenomenon can be obtained
through the use of scaling. When the data of the laser’s power is plotted against the signal from the photodiode received by the amplifier, a slope of the graph is given by the Matlab code. It can be stated that the 1PA or 2PA process is occurring in our experiment if the experimental slope of the Matlab plot is equal to the theoretically predicted value of the slope. For example, if the theoretical model predicts a slope of one for a single photon absorption and the experimental slope is one, then we can be assured that 1PA is happening inside the photodiode. Knowing for certain that the phenomenon is happening, we can proceed to calculate our absorption coefficient. Using the equations for 1PA and 2PA that were derived in previous sections, we can theoretically deduce what the value of the slope should be. If the phenomenon is occurring inside the GaP photodiode, then our theoretical value for the slope should be approximately close to the experimental value generated in the log-log plots of Matlab.

This verification can be done by plotting the natural log of both the voltage from the amplifier as well as the average power, leading to the production of a graph of $\ln(v)$ vs $\ln(P_o)$. The log-log plot was used in order to confirm the existence of 1PA and 2PA in the photodiode because the relationship between photons of the laser and the conduction electrons being generated follow a power-law dependence. One quantity varies as a power of the other quantity. For example, one electron is generated for every two photons in a 2PA process, $i \sim p^2$, where $i$ is the current and $p$ is the photon. The power-law dependence allows us to re-write an equation in a $y = mx + b$ format using natural log so that we can theoretically predict the slope associated with our derived equations. The natural log is simply used in order obtain a $y = mx + b$ format where the theoretical value for $m$ can be seen. This theoretically predicted slope can be compared to the experimental slope in order to verify that 1PA or 2PA processes are occurring within the GaP photodiode.

It is the experimental slope of the log-log graph of our data that is of interest since that is what will be compared to the theoretical slope. First, the theoretical slope for a single photon absorption is derived so that it can be compared. The theoretical model should be able to predict the slope of a $\ln(v)$ vs $\ln(P_o)$ generated by the Matlab program. Equation 1.8 from the prior section for single-photon absorption is given below and is used as the starting point of the derivation,
\[
\frac{i}{P_o} = \left( \frac{\lambda}{hc} \right) az.
\]

In order to theoretically predict the slope, it would be beneficial to arrange the equation into the format of \( y = mx + b \) in order to see what the value of \( m \) would be. This can be done by taking the natural log of both sides,

\[
\frac{G_v}{P_o} = \left( \frac{\lambda}{hc} \right) az
\]

and

\[
\ln \left( \frac{v}{P_o} \right) = \ln \left( \frac{\lambda}{Ghc} az \right).
\]

Some basic simplifications yields

\[
\ln (v) - \ln (P_o) = \ln \left( \frac{\lambda}{Ghc} az \right)
\]

and

\[
\ln (v) = \ln (P_o) + \ln \left( \frac{\lambda}{Ghc} az \right).
\]

With \( y \) being equal to \( \ln(v) \) and \( x \) being equal to \( \ln(P_o) \), the theoretical slope should be \( m = 1 \) for a plot of \( \ln(v) \) vs \( \ln(P_o) \). Therefore, the experimental slope given by Matlab should yield a value of approximately one for the data in continuous wave mode. An experimental slope of one should be expected for 1PA process according to the theoretical value.

In order to see if the chosen experimental set up is compatible with two-photon absorption, a similar theoretical model is developed. The slope of the \( \ln(v) \) vs \( \ln(P_o) \) graph is predicted using the equations derived in prior sections for 2PA. The same procedure involving natural log is exploited in order to find the theoretical slope that can be used to compare to the experimental value. Below is the derivation for the theoretical slope of a 2PA process,

\[
i = \left( \frac{\lambda \beta z}{2hc} \right) \left( \frac{P_o^2}{\pi w_o^2} \right)
\]

and

\[
\frac{G_v}{P_o^2} = \left( \frac{\lambda \beta z}{2hc} \right) \left( \frac{1}{\pi w_o^2} \right).
\]

When the natural logs are taken on both sides, we get

\[
\ln \left( \frac{v}{P_o^2} \right) = \ln \left( \frac{\lambda \beta z}{2Ghc} \frac{1}{\pi w_o^2} \right).
\]

Some algebra can be used to simplify the equation,
\[ \ln(v) - \ln\left(P_p^2\right) = \ln\left(\frac{\lambda \beta z}{2G\hbar e} \frac{1}{\pi w_o^2}\right) \]

and

\[ \ln(v) - 2 \ln(P_p) = \ln\left(\frac{\lambda \beta z}{2G\hbar e} \frac{1}{\pi w_o^2}\right) \]

and

\[ \ln(v) = 2 \ln(P_p) + \ln\left(\frac{\lambda \beta z}{2G\hbar e} \frac{1}{\pi w_o^2}\right). \]

As can be seen from the rearrangement of the equation, the theoretical model predicts the slope to be \( m = 2 \) for the pulsed mode data and two-photon absorption. Consequently, the plot from Matlab should produce a slope of two when the experimental data is graph in the software.
2.1 Experimental Layout

The primary purpose is to experimentally determine $\beta$. This is accomplished by collecting data associated with 1PA and 2PA processes and generating plots of the GaP Signal vs Incident Power. The upcoming sections discuss the methodology employed in the collection of our data as well as introducing the equipment that was vital in the research. Our discussion will encompass the preparation of the equipment and the analysis of the data with its respective plots, finally concluding with the calculation of $\beta$ for our work. We begin our discussion with the experimental layout.

The experimental layout consists of a neutral density filter (NDF), waveplate (WP), polarizing beam splitter (PBS), another beam splitter (BS), and a lens (L) that focuses the laser into the GaP photodiode (PD) as can be seen in Figure 2.1. The first component that the laser encounters is the neutral density filter whose main function is to set a maximum potential intensity for the laser. The intensity of the laser has to be decreased by NDF in order to accommodate the wavelength range that the waveplate can tolerate. With the maximum power of the laser set, the light beam travels through the waveplate before entering PBS. The primary goal of these elements is to alter the intensity of the laser over a certain range.

The laser is already polarized; however, the light beam changes polarization as it travels through the waveplate according to Malus's law. The light beam that emerges out of the waveplate is split into its vertical and horizontal components by the PBS. One component goes in the forward and the other goes in the sideways. Please see Figure 2.1 in order to visualize this process. This phenomenon aides in the altering of the intensity of the laser since the angle of the waveplate determines the polarization; therefore, it determines the amount of power that is allocated into the two directions as the light beam splits. For example, if the waveplate releases a light beam that carries 100 percent of its power in the vertical component then the intensity of the laser would
be at its maximum. All the light beam would travel forward and none of the power would be allocated in the sideways direction. If the waveplate releases a light beam that carries all of its power in the horizontal component then all of the intensity of the beam would be allocated in the sideways direction, making the intensity at its minimum since little intensity would go in the forward direction. The angle of rotation of the waveplate combined with the PBS allows the intensity of the laser to be varied.

From the PBS, the light beam travels into another beam splitter. BS is structured to take approximately 10 percent of the light beam and send it to a power meter where it can be recorded. This is the data associated with the channel zero of the amplifier also known as the incident power, $P_o$. The remaining 90 percent of the light beam goes through a lens where it is focused into the GaP photodiode. After the 1PA and 2PA processes take place, the information from the photodiode is recorded as channel one also known as the GaP signal.

Since the excitation of the electrons in the GaP crystal depends on the power of the laser, it is necessary to have a device that allows for the intensity of the laser to be varied. For this function, a half-waveplate is utilized. This waveplate is mounted on a motor that is capable of rotating it through 360 degree, allowing the intensity of the laser to be varied. The half-waveplate along with the stepper motor is designed, assembled, and tested specifically for these measurements which is why a significant portion of the discussion is allocated to this device. Figure 2.1 below shows both an image of the experimental layout along with an illustration on how the components function together to complete the goal of focusing the laser beam towards the GaP photodiode.

![Figure 2.1. Experimental layout photograph (left) and a schematic layout (right). A laser beam first goes through a neutral density filter (NDF) and then a half-waveplate (WP). From the waveplate, the laser beam enters a polarizing beam splitter (PBS) which splits the beam into two directions. One of the directions is blocked by a beam block (BB). The other direction of the beam travels into another beam splitter (BS). 10% of the beam from this new beam splitter is sent into calorimetric power meter (CPM) where the incident...](image-url)
power is measured and recorded. The other 90% of the beam travels into GaP photodiode (GaP PD). The current generated within the photodiode is recorded and stored as well.

The laser emits a continuous train of very short pulses at a repetition frequency of ~100 MHz and a pulse width of 100 fs into the layout on the table seen in Figure 2.1. The laser beam first encountered an alignment mirror. Once the beam is steered properly, it passes through with a neutral density filter wheel which reduced the laser power to a safe maximum level. A second mirror redirects the beam exiting the ND filter wheel and sends it to the waveplate. Following the waveplate, the light enters a polarizing beam splitter cube. One of the directions was perpendicular to the initial direction of the propagation and was not important for data collection. Because of this, the light traveling in the perpendicular direction was blocked by a beam blocker as seen in Figure 2.1.

Once the intensity has been adjusted a ~10% fraction is diverted by a second, non-polarizing beam splitter into a calorimetric power meter. As can be seen from the illustration, this light exited the cube and proceeds into another beam splitter. This new beam splitter took approximately 10 percent of the light and sent it into a calorimetric power meter which recorded how much power navigated through it. The significant direction of the split beam was the light that passed straight through the cube. The data collected from the calorimetric power meter was sent to the channel zero of the voltage amplifier. This measurement was calibrated to the incident laser power on the sample. The remaining ~90% of the laser power was focused by a lens into the GaP photodiode. The lens is an anti-reflection coated lens that transmits majority of the power; therefore, very little power is lost between the calorimetric power meter compiling channel zero data and the photodiode that is compiling the channel one signal. The signal from the GaP photodiode was sent into an adjustable gain current amplifier (Model SR570). The current amplifier outputs a voltage that was recorded.

2.1.1 Waveplate

A waveplate is an optical device also known as a retarder which has a function of converting one form of polarized light into another form of polarization. Waveplates can be used, for example, to convert a beam of light that is plane polarized into a beam of light that is circularly or elliptically polarized, or vice versa. The
waveplate accomplishes this by taking the incident light and breaking it down into its two components: parallel and perpendicular to the optical axis. One of the components of light is slowed down relative to the other component, meaning a phase difference is created between the two components. The component of the light that travels slower than its partner is deemed to be on the slow axis, while the faster component resides on the fast axis of the waveplate. Upon exiting the waveplate, the components of light are recombined with their new phase difference present. As an example, a quarter-waveplate generates a 90 degree phase difference between the components, causing the transformation of a linearly polarized light into a circularly or elliptically polarized light. The same plate can also convert circular or elliptical form of polarization into a linear form as well. Similarly, if one uses a half-waveplate then one will produce a phase difference of 180 degrees; therefore, waveplates do change the polarization of the incident beam but not all do it in the same manner or with the same results (Stanley, 1968). The type of waveplate used should reflect the type of polarization desired.

Figure 2.2. An illustration of a waveplate’s interaction with light. An incident light that is linearly polarized enters a quarter-waveplate element. After exiting the optical device, the light is now circularly polarized due to the phase difference between components generated by the optical device.

The waveplate utilized in the experimental layout presented in Figure 2.1 is a zero order quartz waveplate made of crystal quartz with a diameter of 12.7 mm. It is often used with femtosecond lasers, such as ours, that have wavelengths in the range of UV through infrared. Waveplates can come in different orders and our waveplate is in the zero order. Multi-order waveplates have a much larger phase change than the required value. This differs from zero order waveplates where the phase change is the direct value. For example, the phase change would be exactly $\pi$ for a half waveplate of zero order; whereas, the multi-order waveplate would produce a phase delay that is $\pi$ plus some integer, causing a much larger phase delay.
Given below in Figure 2.3 is the diagram representing the elements that constitute our waveplate. Waveplates, in general, are constructed of material that exhibits birefringence, meaning the material displays two different indices of refraction. Materials with properties of birefringence can interact differently with lights of different polarization. For example, one type of polarization can be nearly transparent while another direction of polarization can be absorbed by the material. These crystals do not have optical properties that are same in all directions. These birefringent crystals have two different indices of refraction that allow the light to be separated into parallel and perpendicular components where light that encounters a lower index of refraction will travel along the fast axis at a faster speed than its counterpart. This phenomenon is represented by the relationship of \( c/n = v \) where \( c \) is the speed of light and \( n \) is the index of refraction. For a birefringent crystal such as quartz, the two different indices of refraction are 1.5443 and 1.5534 (Hecht, 1987).

![Figure 2.3. Picture of the waveplate (left) utilized in the experimental set up obtained from Idex Optics & Photonics (Part Number QWPO-800-08-2-R10). Image of a diagram of the components (right) that constitute the waveplate. As can be seen, the waveplate consists of two quartz crystals whose optical axes are crossed in order to generate a phase difference due to a change in index of refraction.](image)

Within the experimental set-up, the waveplate was mounted onto a stepper-motor that allowed for the waveplate to be rotated through various angles. The waveplate was rotated during the experiment in order to adjust for the power of the light which was later measured by the calorimetric power meter. The voltage recorded from channel zero of the current amplifier was the power of the laser attenuated due to the rotation of the waveplate. This aspect of physics can be clarified through the use of the Malus’s law. Malus’s Law is given by the relationship of \( I(\theta) = I_o \cos^2(\theta) \) where \( I(\theta) \) is the final intensity being emitted through the optical device, \( I_o \) is the initial intensity of the light, and \( \theta \) is the angle by which the transmission axis is shifted. Only the
Within the experimental set-up, the waveplate was mounted onto a stepper-motor that allowed for the waveplate to be rotated through various angles. The waveplate was rotated during the experiment in order to adjust for the power of the light which was later measured by the calorimetric power meter. The voltage recorded from channel zero of the current amplifier was the power of the laser attenuated due to the rotation of the waveplate. This aspect of physics can be clarified through the use of the Malus’s law. Malus’s Law is given by the relationship
\[ I(q) = I_0 \cos^2(q) \]
where \( I(q) \) is the final intensity being emitted through the optical device, \( I_0 \) is the initial intensity of the light, and \( \theta \) is the angle by which the transmission axis is shifted. Only the component that is parallel to the transmission axis of the polarizer or waveplate will pass through the optical device undisturbed. As the polarizer or the waveplate is rotated manually, the angle will change between the transmission axis of the polarizer and the polarized incident light. As our waveplate is rotated by the stepper motor, the angle between the polarized laser beam and the transmission axis of the waveplate will change. This will alter the intensity of the light that exits the waveplate. Since intensity is power per surface area, the calorimetric power meter will change its readings of power according to the change in angle. Therefore, the power detected by channel zero of the current amplifier is related to the orientation of the angle of the waveplate. In order to understand Malus’s law further, an example will be employed. There are two polarizers: one horizontal and one vertical. Natural light, which is unpolarized, is incident on the horizontal polarizer. The light that emerges out of the horizontal polarizer will be linearly polarized in the horizontal direction. If the horizontally polarized light enters the vertical polarizer, the final intensity would be zero according to Malus’s Law. Since the light is horizontally polarized and the axis of transmission for the vertical polarizer is in the vertical direction, the angle between the two is 90 degrees. The following calculation would be obtains an intensity of zero:
\[ I(\theta) = (I_0)\cos^2(90) = 0. \]
The horizontal and vertical polarizers are said to be crossed with each other since the horizontal polarizer produces a light that is polarized in the direction that is 90 degrees from the transmission axis of the vertical polarizer. The same interaction transpires when our polarized laser comes in contact with a waveplate that is rotating through various angles with the assistance of the stepper motor.

### 2.1.2 Stepper Motor

As described in prior section, the waveplate was rotated through various angles in order to produce an intensity that varies. This allowed the non-constant power of the light to be plotted in a graph against the data collected from the photodiode regarding the current produced through 1PA or 2PA phenomenon. The rotation of the waveplate was created by mounting the device on a rotary table that was motorized by a stepper motor. The information regarding the device can be obtain from Velmex website (http://www.velmex.com/Products/Rotary_Tables/Motorized-Rotary-Tables.html). The machine can be used to
As described in prior section, the waveplate was rotated through various angles in order to produce an intensity that varies. This allowed the non-constant power of the light to be plotted in a graph against the data collected from the photodiode regarding the current produced through 1PA or 2PA phenomenon. The rotation of the waveplate was created by mounting the device on a rotary table that was motorized by a stepper motor. The information regarding the device can be obtained from Velmex website (http://www.velmex.com/Products/Rotary_Tables/Motorized-Rotary-Tables.html). The machine can be used to rotate mirrors, cameras, or sensors up to 600 rpm by attaching the objects to the platform of the motor. Below is an image of the rotary table, unaltered by our modifications.

![Image of Velmex rotary table](http://www.velmex.com/Products/Rotary_Tables/Motorized-Rotary-Tables.html)

Figure 2.4. An image of Velmex rotary table that is used to rotate an object that is mounted on its stage. The golden metal seen in the pictures has four screw holes in order to tighten the object to the rotation stage. For this situation, the object that was mounted was a waveplate. The circular black plastic with white markings of angles is the portion of the device that physically rotates with the assistance of the stepper motor. The stepper motor is seen behind the rotation stage with several wires exiting out of its side. The wires are wrapped in black electric tape and are attached to a white connector. These wires allow current to flow into the stepper motor and through the coils that generate the magnetic field responsible for the rotation of the stage. Picture obtained from Velmex website. (http://www.velmex.com/Products/Rotary_Tables/Motorized-Rotary-Tables.html).

The rotary table can either be manually turned by a knob or can be rotated by a motor. The motorized rotation of the device is accomplished through the use of a stepper motor. The stepper motor inside the device functions by utilizing magnets and electromagnetism. Stepper motors come in many varieties which can be a motor driven by permanent magnets or driven through variable reluctance. In a lot of instances, the motors are hybrids that are engineered for specific functions. It is not possible to known with complete certainty what type of motor is inside our rotary table since the information from Velmex does not specify and our rotary table was not disassembled for examination. However, it is known that it is a stepper motor. Inside the motor, two main components exist that are essential: an outer ring and a center shaft. There is a shaft in the center of the machine that rotates in a circle when the motor is turned on. This segment has “teeth” on its surface and permanent magnets embedded inside its structure. Figure 2.5 below shows a diagram of the center shaft. How many teeth the motor has varies from one model to the next since the teeth are the steps that the motor takes as it rotates. Those motors with smaller steps are going to have more teeth, and greater precision in their movement.
Figure 2.5. An illustration of the center shaft that is found inside the motor. The center shaft has permanent magnets embedded inside its structure and “teeth” as seen in location A of the illustration. The teeth of the shaft allow motion to transpire since the teeth function as gears. Location B on the illustration shows the axis of rotation for the shaft. As stated, the shaft contains magnets and these magnets will feel an attraction to other magnets in its surrounding environment. It is this attraction that is exploited in order to move the shaft in a circle since a force will be generated due to the attraction, hence motion will take place. The physics of magnetism is exploited in order to create circular motion.

The center shaft of the machine is surrounded by a larger ring that assists the shaft in rotating as seen in Figure 2.6. This ring also possess “teeth,” and these teeth are capable of locking with the teeth of the center shaft like a pair of gears on an old wind-up wristwatch. When one of the teeth from the ring is magnetized by the flow of the current, it will attract a tooth from the center shaft that is opposite in polarization than itself. The magnetization of the ring is caused by flowing a current through a pair of wires seen in Figure 2.6 where locations A, B, A’, and B’ represent the different coils of wires.

Figure 2.6. An illustration of the inside of a stepper motor. The center shaft can be seen inside the ring with both of the components containing their separate teeth. The coils through which the current flows are represented by A, B, A’, and B’ in the diagram. When the L293D chip allows current to flow, the coils generate a magnetic field that acts like a magnet which attracts the center shaft, causing motion. When the L293D chip blocks the flow of current, the coils generate no magnetic field and no attraction takes place, causing no motion.
In order to cause the teeth of the ring to become magnet-like, the coils of the wires are used to generate current. In the subsequent section of 2.2, a discussion was had about the multiple wires that emerge out of the motor. These wires can also been seen in Figure 2.4. The wires were cut and a multi-meter was utilized in order to figure out which two wires form a closed current. The two wires, which allow a current to flow successfully, are wires that are attached to the same coil. The wires carry a current into the coils which generate a magnetic field that attracts the permanent magnets of the center shaft. When a current is run through the two partner wires and a magnetic field is generated, the attraction between the shaft and the ring cause a rotating motion. When the coils behave like a magnet with a south-pole, the ring teeth will attract the teeth on the center shaft that have north polarity. The teeth will lock to each other due to their attraction. In which direction the current flows, or whether the current flows at all, is controlled by the L293D chip mention in the upcoming section of 2.2.

In the above Figure 2.6, the L293D chip will allow current to flow through the coils in location A of the illustration while blocking the current flow to the other coils. Since location A will be the only one with a current flow, the location will have a magnetic field that will attract the teeth on the center shaft. The shaft will move towards location A. When the Matlab code instructs the L293D chip to block the flow of current into the coils of location A, the location will no longer have a magnetic field and there will be no attraction. Then, the L293D chip will allow the current to flow into the coils of location B, generating a magnetic field in that location. Now the teeth of the shaft are attracted to location B and will rotated towards it, allowing the shaft to move. The codes that drive the stepper motor are written in a manner that allows each coil to be turned on in a sequence. The first location turned on by the chip could be location A followed by B then A’ and B’ which causes the shaft to turn itself in a circle. In order to cause the stepper motor to move in the reverse direction, the sequence can be run back words from B’ to location A. Therefore, the stepper motor can move clockwise or counterclockwise, depending on which set of coils is turned on first by the code.
2.2 Preparation of the Equipment

Before the experimental data was collected, the equipment had to be properly prepared. A method had to be devised where the waveplate could be mounted on the rotary device, allowing it to be rotated a full circle. This apparatus that housed the waveplate was designed via SCAD software and printed out using a 3D printer. Eventually, the housing was secured to the rotation stage of the rotary table along with the waveplate. Other modifications had to be made to the rotary table as well in order to make it functional. An electric circuit board had to be constructed that was able to control the stepper motor from a computer, and this portion was assembled on a breadboard utilizing a L293D chip and a Raspberry Pi GPIO. Within this section, a discussion will be presented regarding the methodology and the execution of these goals.

2.2.1 Examination of the Stepper Motor: Understanding its Structure and Properties

The stepper motor functioned as a holder of the waveplate that was used during the collection of data. Along with being attached to the waveplate, the stepper motor allowed it to be rotated through different angles which altered the power of the laser beam. Figure 2.7 presents an image of the final outcome: a stepper motor with an attached waveplate.

![Figure 2.7](image)

Figure 2.7. The rotary table is shown in the picture in both modified and unmodified states for comparison. The orange housing attached to the rotary table was constructed by SCAD software in order to hold the waveplate in place as the stage rotated through different angles. The orange housing was designed via SCAD and was printed out using a 3D printer. The orange housing was then attached to the stage of the rotary table using screws. Below the rotation stage and the orange housing is the stepper motor with its wires protruding outwards.

Before any modification was done to the stepper motor, the motor was examined in order to get a greater understanding of how it functions. The motor’s structure and properties had to be explored without destroying
the device since very little information regarding the stepper motor was provided by the manufacturer Velmex. There was not a significant amount of knowledge regarding the switch mechanism inside the motor since the product’s manual did not discuss this topic in great detail, making it difficult to construct a breadboard that controls the motor when little is known.

![Figure 2.8](image)

**Figure 2.8.** A picture of the stepper motor without any modifications. (a) Rotation stage where the orange housing containing the waveplate was attached with screws. This portion of the device moves in a circle when the motor is turned on. (b) Stepper motor which causes the rotation of the stage. (c) Wires that carry the current which powers the stepper motor and allows the coils of the motor to produce magnetic fields. (d) Knob used to manually operate the motor if such action is desired.

The components of the unmodified stepper motor include the rotation stage where a waveplate will be mounted, a stepper motor that drives the stage, and a set of six wires that exit the motor which supply it with a current. As can be seen in Figure 2.8, the six wires are held together by a black protective material that connects the white plastic housing and the stepper motor. Within the white housing, each wire has its own hole and its own respective metal pin as pictured in Figure 2.9. The six different wires can be distinguished from one another through their different colors which consisted of black, green, pink, white, blue, and yellow. Through the examination of the motor, it was concluded that the wires provide different services to the motor and a close up of the wires can be seen below.
In order to figure out how each of the wires is related to the switch, a multimeter was used to determine at what conditions the switch inside the motor is closed. Since a current flows only when a circuit is complete, the “continuity” feature on the multimeter was utilized to locate a closed circuit as the motor rotated full 360 degrees. Wires had their respective partners. And when a voltage was applied to both of the partner wires, a current was generated through the closed circuit: a battery, a wire, a coil, and its respective partner wire. By attaching clips of the multimeter to different ends of each wire, it was found that the black wire and the green wire formed a closed circuit at specific range of angles. Along with black and green, it was found that the pink and blue wires also had the same relationship.
Figure 2.10. A breadboard contains two green terminals. These terminals contain all six wires that protrude from the stepper motor. Using a spare blue wire seen in the picture, a circuit can be created on the bread board. Different colored wires were paired with each other to see which partnership produced a closed circuit. A closed circuit indicated that the two wires were connected to the same coil. The motor was cranked by its knob using a wrench to see if there was a physical resistance to the rotation of the wrench. A presence of resistance was an indication of current flow and, therefore, a successful pairing of the wires. Those two wires carried the current to the coils, allowing a magnetic field to be generated.

Alternatively, another method was used to verify the partnership among the six wires. The plastic housing was removed, leaving the bare wire as can be seen in Figure 2.9. The bare wire was placed inside the green terminals that were plugged into a breadboard. This set-up can be seen in Figure 2.10 above. A spare wire was used to connect the terminal that contained the pink wire with the terminal of the blue wire in order to create a closed circuit. A wrench was used to manually rotate the knob of the motor since the presence of resistance is an indication that there is a coil between the two wires. As the knob was rotated, a physical resistance was observed in those wires that were paired with each other. Pink and blue wires had a discernible resistance when paired together, indicating the presence of a current flow. Besides the pairing of black and green, other wires exhibited either weak or no resistance in their partnership.

The knob is harder to turn for those wires that are partners whose pairing forms a closed circuit. This is caused by the structure of the stepper motor that is responsible for rotating the stage. Inside the stepper motor is a set of coils like a solenoid. Each partner wire is connected to one end of the coil and together form a closed circuit. This leads to electromagnetic induction. The stepper motor has a center shaft that contains permanent magnets. As the knob manually rotates the stage, these magnets are in motion. A magnet in motion creates a changing flux. The changing flux through the coil induces an electromotive force and produces a current in the coil. This induced current produces its own magnetic field that attempts to resist the change in the system. The
direction of the induced magnetic field and the induced current will be in the direction that oppose the change that created it. Therefore, a resistance is felt when the knob is being turned that makes it difficult to move the permanent magnets located inside the shaft of the stepper motor. Through this process, it can be determined which wires form a closed circuit.

2.2.2 Construction of the Electronics that Control the Stepper Motor

A breadboard was constructed for the main purpose of connecting the stepper motor to the Raspberry Pi software within the computer. The software allows the stage of the motor to rotate by utilizing a specific set of Matlab codes. The codes turn the GPIO (General Purpose Input Output) pins on the Raspberry Pi on or off, depending on the input from the user. The Raspberry Pi pin will send a message to the L293D chip regarding its activation, and the chip will either release or block current based on that information. If the current is released, it will exit the output of the L293D chip and flow into the motor, causing the motor to run. Therefore, the three main components of the breadboard are the L293D chip, the Adafruit T-Cobbler Plus chip for Raspberry Pi, and the six wires that protrude from the motor.

![Figure 2.11](image)

*Figure 2.11. A breadboard which contains the green terminals that house the six wires of the motor, the L293D chip, and the Adafruit T-Cobbler Plus chip which is necessary for the use of the Raspberry Pi software within the computer.*

The L293D chip is a device that is commonly used with a stepping motor, and it is a quadruple high current half-H driver. A driver in electronics is a device that controls or regulates another device, component, or circuit. In this particular situation, the L293D driver is designed to drive inductive loads such as stepping motors which
The L293D chip is a device that is commonly used with a stepping motor, and it is a quadruple high current half-H driver. A driver in electronics is a device that controls or regulates another device, component, or circuit. In this particular situation, the L293D driver is designed to drive inductive loads such as stepping motors which require a large amount of current and can produce a large voltage when turned on. This driver provides a current of up to 600-mA at a voltage range of 4.5 to 36 volts. In Figure 2.12, we are given a schematic of the L293D chip.

**Figure 2.12.** The L293D chip has two pins designated for controlling whether the current will be released, pins 1.2EN and 3.4EN. Pin 1.2EN enables drivers 1 and 2, and pin 3.4EN enables drivers 3 and 4. These two pins determine whether the current will pass or be blocked based on the input it receives. When the pins 1.2EN or 3.4EN receive high electronic input from the computer, the associated drivers 1A, 2A, 3A, and 4A are turned on and their respective outputs 1Y, 2Y, 3Y, and 4Y become active. If the pins 1.2EN and 3.4EN receive low input from the computer, their drivers become disabled and their respective outputs become inactive. If the drivers and the outputs are enabled and active, a current flows through the circuit which can be up to 600-mA for a L293D chip.

The L293D chip is constructed using H-bridge system where transistors are used as switches to be turned on and off. This arrangement is central to causing the motor to move forward and backwards depending on what electrical input is given into the chip. The electrical inputs are in the form of 1 being on and 0 being off. H-bridge uses four transistors/switches that will be activated using electrical signals, choosing which pair of switches will be turned on or off will determine the direction of the motor’s motion. The selection of switches that will be turned on or off will determine in which direction the current will flow through the circuit. This gives the L293D chip its unique property of having bidirectional drive currents.

The three components that constitute the breadboard were connected by wires as can be seen in Figure 2.14. The set of green wires in the photo connects the “A” outputs of the L293D chip to the Raspberry Pi chip (Adafruit T-Cobbler Plus chip). To familiarize yourself with what the “A” outputs are of the L293D chip, please refer to Figure 2.12 for more information regarding the schematics of the chip. When the pins 1.2EN receive high electronic input from the Raspberry Pi GPIO, the associated drivers 1A and 2A are turned on and
their respective outputs 1Y and 2Y become active. If the pins 1,2EN receive low input from the computer, their drivers become disabled and their respective outputs become inactive. If the drivers and the outputs are enabled and active, a current flows through the circuit which can be up to 600-mA for a L293D chip. This current that was permitted to flow out of 1Y or 2Y will eventually reach the coils in the stepper motor, allowing rotation of the stage. As can be observed in Figure 2.14 given below, the Raspberry Pi GPIO has its own set of inputs and outputs that can be programmed using Matlab codes to communicate information to chip L293D.

![Figure 2.14](image_url)

**Figure 2.14.** Close-up photo of the Raspberry Pi GPIO (Adafruit T-Cobbler Plus chip). The sides of the chip contain input and output pins that can be programmed using Matlab codes. For example, the marking “GND” represents the ground and “#23” represents pin 23. These pins accept information from the software and transmit that information throughout the breadboard.

For the sake of transparency, a table is inserted to show how each pin of the L293D chip was connected to the Raspberry Pi GPIO. The table below demonstrates how each "A" output of L293D chip was joined to the outputs of Raspberry Pi chip. For example, 1A output of L293D chip is connected by a green wire to the output Pi #22 of Raspberry Pi GPIO. For an illustration of the breadboard set-up, please refer to Figure 2.15 at the end of the section.

<table>
<thead>
<tr>
<th>L293D Inputs</th>
<th>Raspberry Pi GPIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 A</td>
<td>Pi #22</td>
</tr>
<tr>
<td>2 A</td>
<td>Pi #23</td>
</tr>
<tr>
<td>3 A</td>
<td>Pi #24</td>
</tr>
<tr>
<td>4 A</td>
<td>Pi #25</td>
</tr>
</tbody>
</table>

**Table 2.1.** A table that represents how the breadboard is connected with the use of wires. It shows the relationship of the L293D chip and the Raspberry Pi GPIO. One end of the wire is placed in the terminal of the L293D chip, and the other end of the wire is placed in the terminal of the Raspberry Pi chip. For example, the table states that the 1A input of the L293D chip is connected with a wire to the pin #22 of the Raspberry Pi on the breadboard.
A similar process is utilized when relating the other inputs of the L293D chip, such as the enable pins. The two blue wires within the photo represent the connection between the EN outputs of the L293D chip to the respective outputs of the Raspberry Pi GPIO as given in Table 2.2 below.

<table>
<thead>
<tr>
<th>L293D Inputs</th>
<th>Raspberry Pi GPIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>EN 1,2</td>
<td>Pi 5V</td>
</tr>
<tr>
<td>EN 3, 4</td>
<td>Pi 5V</td>
</tr>
</tbody>
</table>

**Table 2.3.** A table that represents the relationship between the pins of the L293D chip with the pins of the Raspberry Pi GPIO. For example, the EN12 output of L293D chip is connected by a blue wire to the Pi 5V output of the Raspberry Pi GPIO. The 5V representing 5 volts which were provided by a power supply to the breadboard. The yellow wires in the photo in Figure 2.11 connect the Vcc1 and Vcc2 to the external 5V power source while all the ground outputs of the L293D chip are connected to the ground outputs of the Raspberry Pi chip by orange wires.

Finally, the last remaining pins of the L293D are given in the table below with their relationship to the Raspberry Pi GPIO. The colored wires stated in Table 2.3 relate to the wires that emerge out of the stepper motor. These wires are locked into a green terminal, and this terminal is inserted into the breadboard in order to allow current to flow into the stepper motor. Another set of yellow wires, not mentioned in any of the Tables, were used to connect the Vcc1 and Vcc2 of the L293D chip seen in Figure 2.12 to an external 5V power source. Once again, please refer to Figure 2.15 for a schematic of the breadboard layout.

<table>
<thead>
<tr>
<th>L293D Inputs</th>
<th>Raspberry Pi GPIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Y</td>
<td>Black wire</td>
</tr>
<tr>
<td>2 Y</td>
<td>Green wire</td>
</tr>
<tr>
<td>3 Y</td>
<td>Pink wire</td>
</tr>
<tr>
<td>4 Y</td>
<td>Blue wire</td>
</tr>
</tbody>
</table>

**Table 2.5.** A table that represents the relationship between the pins of the L293D chip with the pins of the Raspberry Pi GPIO. The red wires in the photo of Figure 2.11 represent the connection between the “Y” outputs in the L293D chip and the green terminals. The green terminals contain the six wires from the motor. For example, output 1Y of the L293D chip is connected by a red wire to the green terminal which contains the black wire from the motor.

The entire connection on the breadboard between the three components allows the Matlab program to rotate the stage of the motor when necessary. The Matlab code sends information to Raspberry Pi GPIO pins, determin-
The entire connection on the breadboard between the three components allows the Matlab program to rotate the stage of the motor when necessary. The Matlab code sends information to Raspberry Pi GPIO pins, determining whether the enable pins (“EN”) of the L293D chip should be turned on. If turned on, then the enable pins activate their respective drivers (“A”) which let currents flow from their outputs (“Y”). This current travels through the breadboard and into the wires housed in the green terminals. From there, the current reaches the stepper motor that makes the stage rotate. It permits a greater variety of use for the waveplate that was engineered to be mounted on the rotation stage of the stepper motor.

![Diagram of breadboard connections](image)

**Figure 2.15.** An illustration of the layout of the breadboard. The arrows represent different wires on the breadboard used to connect different electronic components. The green terminal hold the wires from the stepper motor and allow those wires to be plugged into the breadboard. The Raspberry Pi GPIO receives the directions from the Matlab code. Meanwhile, the L293D controls the current flow.

### 2.2.3 Engineering of the Housing for the Waveplate using SCAD software

The waveplate is required to be positioned on top of the stage of the stepper motor since the light of the beam will travel into the waveplate and through the center hole of the rotation stage. But in order to do so, a mount must be designed to hold the waveplate in place. A 3D printer was used to print out the plastic housing that the SCAD software designed for the waveplate seen below in Figure 2.16.

![Plastic housing for waveplate](image)

**Figure 2.16.** Plastic housing for the waveplate is mounted on top of the metal portion of the rotation stage using screws. The housing was designed using SCAD software and was printed by a 3D print into three different parts. The three different parts consist of the top and bottom elements which are identical to each other and the middle element which is different. The middle element is shaped like a donut and allows the waveplate to be placed inside the hole of the “donut”. Then, the top and bottom elements of the housing
sandwich the middle component. Screws are placed into the holes. Once these three different parts are screwed into the metal holes of the stage, a housing for the waveplate is created.

The plastic housing has three components that hold the waveplate in place and are screwed into the stage of the stepper motor. The top and bottom of the housing are identical to each other, but the middle portion is slightly different. The primary purpose of the top component is to keep the waveplate from falling out when the motor is held in different positions; meanwhile, the purpose of the bottom component is to prevent the waveplate from being scratched by the friction caused from rotating the stage. The middle component of the housing is the element that locks the waveplate in place and keeps it sandwiched between the top and bottom portions.

![Figure 2.16](image)

**Figure 2.16.** Plastic housing for the waveplate is mounted on top of the metal portion of the rotation stage using screws. The housing was designed using SCAD software and was printed by a 3D print into three different parts. The three different parts consist of the top and bottom elements which are identical to each other and the middle element which is different. The middle element is shaped like a donut and allows the stepper motor. The top and bottom of the housing are identical to each other, but the middle portion is slightly different. The primary purpose of the top component is to keep the waveplate from falling out when the motor is held in different positions; meanwhile, the purpose of the bottom component is to prevent the waveplate from being scratched by the friction caused from rotating the stage. The middle component of the housing is the element that locks the waveplate in place and keeps it sandwiched between the top and bottom portions.

![Figure 2.17](image)

**Figure 2.17.** A coordinate system with the origin in the center of the opening is drawn on the stage. This coordinate system has the units of mm and each of the axis purposely goes through the centers of the screw holes. SCAD software also works with a coordinate system measured in mm. Whatever is measured with calipers on the rotation stage can be directly transferred into the SCAD design since both of the situations have identical coordinate systems.

To fabricate the housing, a code was written using the 3D design program called SCAD to produce each of the cylinder-shaped pieces. The units used for measurement in both SCAD and the 3D printer are in millimeters. Therefore, an arbitrary axis on the rotation stage was drawn that was in the units of millimeters. This axis could be translated into the SCAD software, allowing a connection between the reality of the rotation stage and the virtual design of the software. The software as well as the rotational stage both have the same coordinate system with units of millimeters where the measurements done in one would be equal in the other. Each of the axes ran through one of the holes of the rotation stage that was designated for screws. This was an important distinction since the screw holes for the housing had to perfectly match the positioning of the screw holes of the rotation stage. If the 3D printer did not print the housing that was aligned with the stage, then the screws would not be able to attach the housing to the device. Good alignment begins with properly measuring the holes on the
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Figure 2.18. The photo shows the measurements obtained for the metal ring as well as the opening inside the stage. The radius from the center of the stage to the edge was found to be 20.5 mm. Meanwhile, the calipers measured the radius from the center of the stage to the edge of the opening and found it to be 6.25 mm. This information was used to calculate the coordinate points of the screw holes in the coordinate system previously selected.

By subtracting 5 mm with the radius of the metal ring which was measured to be 20.5 mm, I was able to find out that the screw holes were approximately 15 mm away from the center of the rotation stage. This knowledge of the screw holes was utilized in the SCAD software by designing the holes to be 15 mm in distance from the origin. The location of each of the screws can be seen in the image below with the origin of the coordinate system being located in the center of the stage. Each of the four screw holes was granted its own coordinate point on the axis of the SCAD software as seen in Figure 2.19.

Figure 2.19. The four screw holes each have their own location on the coordinate system with respect to the origin. Since each of the screws was 15 mm away from the center, the coordinate points of the screw holes all contain a value of 15.

This coordinate system was used in SCAD in order to translate 15 mm to each of the four quadrants in order to create holes for the screws. The symmetry of the rotation stage was employed in the creation of the top and
bottom pieces which are both cylinder shaped. These cylinder-shaped top and bottom pieces of the housing were constructed with a large cylinder created on SCAD that had a radius of 20.5 mm. A smaller cylinder that had the radius of 6.25 mm was subtracted from it in order to create an opening in the center of the stage through which the light of the beam can pass. The image of such a cylinder generated by SCAD software can be seen below in Figure 2.20.

![SCAD design of the top and bottom element of the waveplate housing.](image)

**Figure 2.20.** SCAD design of the top and bottom element of the waveplate housing. This SCAD image does not contain the holes for the screws which have not yet been added to the code. It can be seen from the image that the opening through which the beam of the laser will pass through has a radius of 6.25 mm.

Once the general shape of the cylinder housing was constructed, the screw holes were also added into the SCAD design. Small cylinders of radius of 1.75 mm were subtracted from the total image in order to create the screw holes. This allowed the following construction for the top and bottom of the waveplate holders to be generated:

![Final and finished product for the top and bottom portion of the waveplate housing.](image)

**Figure 2.21.** Final and finished product for the top and bottom portion of the waveplate housing. The large opening in the center of the image is for the entrance and exit of the laser beam; meanwhile, the four smaller holes are for the screws which will be used to mount the housing onto the rotation stage.

The entire SCAD code that was utilized in producing the top and bottom of the waveplate holder is given below. Since all three components of the housing are cylinder shaped, the SCAD code uses cylinders in its
The entire SCAD code that was utilized in producing the top and bottom of the waveplate holder is given below. Since all three components of the housing are cylinder shaped, the SCAD code uses cylinders in its design:

```scad
difference()
  {cylinder(3.5, 1.75, 3.5, center=true);
   cylinder(3.5, 2.25, 3.5, center=true);
   translate([25, 0, 0]);
   cylinder(3.5, 1.75, 1.75, center=true);
   translate([-15, 0, 0])
   cylinder(3.5, 1.75, 1.75, center=true);
   translate([0, 15, 0]);
   cylinder(3.5, 1.75, 1.75, center=true);
   translate([0, -15, 0]);
   cylinder(3.5, 1.75, 1.75, center=true);
}
```

**Figure 2.22.** SCAD code written in the SCAD software that designed the top and bottom components of the waveplate housing. Such code produces a cylinder shape housing with a large hole in the center for the passage of light and four tiny holes for screws. The SCAD code is then sent to a 3D printer.

Same process was applied in the creation of the middle piece of the waveplate holder with some minor modifications to the SCAD code. In this instance, the opening inside the cylinder was made bigger in order to be allowed to hold the waveplate whose own radius was larger than the previous openings. The opening inside the middle component of the housing was 12.7 mm in order to comfortably cradle the waveplate. Not only was the radius of the middle component of the housing larger, but the middle piece was also thicker as well. The height of the previous top and bottom elements was 3.5 mm. Meanwhile, for the middle piece the height was raised to 7 mm to properly fit the waveplate whose thickness was approximately 6.35 mm. Figure 2.23 provides the image of the middle element of the waveplate's housing designed through the SCAD software.

```scad
difference()
  {cylinder(7, 1.75, center=true);
   cylinder(7, 1.75, 1.75, center=true);
   translate([15, 0, 0]);
   cylinder(7, 1.75, 1.75, center=true);
   translate([-15, 0, 0])
   cylinder(7, 1.75, 1.75, center=true);
   translate([0, 15, 0]);
   cylinder(7, 1.75, 1.75, center=true);
   translate([0, -15, 0]);
   cylinder(7, 1.75, 1.75, center=true);
}
```

**Figure 2.23.** Image of the middle component of the waveplate housing. The middle component is donut shaped with a large hole in the center in which the waveplate can be inserted into. This opening is just large enough to closely snuggle the waveplate. The design also contains four small holes used for screws that allow all three pieces of the housing to be screwed into the stage. Next to the SCAD image is the code used to create the image.

When the three components of the waveplate housing were screwed into the rotation stage, the waveplate was successfully cradled inside the housing and the stage was capable of rotating without damaging the wave-
plate. This modification to the rotary table allowed the waveplate to be part of the experimental layout without being damaged or destroyed by the process. For more information regarding SCAD, the software can be found on the website: http://openscad.net/.

2.3 Analysis of the Experimental Data

The first step in data analysis is to determine the slopes of Photodiode’s Signal vs. Incident Power for the single photon absorption. These experimental slopes were utilized in calculating the thickness of the GaP crystal inside the photodiode. The thickness of the crystal is necessary in the calculations of the $\beta$ coefficient. And because of this, it is important that a numerical value be known. However, the photodiode arrived from the manufacturer pre-assembled. Without being able to touch or see the crystal, it was impossible to know how thick the GaP material was. Since Thorlabs did not have information regarding thickness, the numerical value had to be obtained through different means. The slopes of the 1PA process were utilized in the determination of the thickness. And once this numerical value was known, the experimental slopes of the 2PA system were used to calculate the $\beta$ coefficient. The experimental $\beta$ was compared to the values of $\beta$ obtained by other publications. The upcoming calculation will begin with the calculation of $z$ followed by the calculation of $\beta$ using the numerical value of $z$.

The analysis of the experimental data will also consist of analyzing the scaling discussed in section 1.3.6. The log-log plots generated of the data are utilized in confirming that there is indeed a single and double photon absorption happening inside the GaP photodiode. It would be pointless to try to obtain an experimental $\beta$ absorption coefficient if there is no 1PA or 2PA process occurring inside the material. Before any calculations are done, it is necessary to be certain that the phenomenon is actually occurring inside the photodiode. This confirmation is done through the use of log-log plots. The theoretical model predicted that the log-log plot of a 1PA process should yield a slope of one, meanwhile, the slope of 2PA process should be two. A comparison between the slopes of the theoretical model with the experimental will be made to see if there is a presence of 1PA and 2PA phenomena inside the GaP crystal of the photodiode.

Next, the graphs generated by Matlab are also analyzed in order to see whether the experimental layout was conducive for good results and whether the data obtained was of quality. Different Matlab graphs were plotted
from 1PA processes to 2PA processes as well as the scaling plots discussed in the prior section of 1.3.6. A discussion of these graphs as well as their residuals is presented in the last section of the thesis as well as the Appendix.

2.3.1 Calculating the Thickness of the GaP Crystal using Data from Single Photon Absorption

The average power of the laser, labeled as channel zero in the voltage amplifier, was measured and recorded. Along with channel zero’s data, the data of channel one was also recorded which corresponds to voltage measured by the amplifier. This voltage, \( v \), is related to the photocurrent, \( i \), inside the photodiode through the gain factor of \( i = vG \). Therefore, the voltage measured by the voltage amplifier in channel one corresponds to the signal inside the photodiode where the relationship is represented by the gain factor. The measured values of channel one were plotted against the measured values of channel zero, producing a plot of Voltage versus Average Power of the laser. This represents the photodiode signal and the incident power of the laser in the continuous wave mode for single photon absorption. The slope of this graph, \( v \) vs \( P_o \), is an important component of the equation for single-photon absorption previously derived in the theoretical model. The equation for the process is given below by Equation 1.8 from Section 1.2.4,

\[
i = \frac{I}{hc} P_o a z.
\]

As can be seen, the slope, \( m \), is equal to \( m = i / P_o \) and is a significant part of the 1PA equation. Due to this fact, the slope that is calculated, through the use of the data obtained in the lab, is necessary in the calculation for our \( z \) value. Of course, the \( z \) value being the thickness of the GaP crystal which needs to be known in order to calculate \( \beta \). The "1PA equation" that was theoretically derived for single-photon absorption is given below in terms of the experimental slope,

\[
\frac{i}{P_o} = \frac{\lambda}{hc} a z.
\]

Inserting the slope, \( m \), into the equation, we get

\[
m = \frac{\lambda}{hc} a z.
\]

The numerical value for \( m \) was acquired from laboratory measurements. Four trials were done, and four set
of measurements were obtained for both the average power of the laser as well as the voltage from the amplifier. From these four sets of data, four slopes were generated from each respective plot.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Slope (V/V)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.079</td>
<td>.005</td>
</tr>
<tr>
<td>3</td>
<td>2.080</td>
<td>.004</td>
</tr>
<tr>
<td>5</td>
<td>2.062</td>
<td>.003</td>
</tr>
<tr>
<td>6</td>
<td>2.063</td>
<td>.003</td>
</tr>
</tbody>
</table>

Table 2.7. Table of the slopes obtained from the continuous wave data for single-photon absorption in GaP material of the photodiode. Four trials resulted in four different sets of data. From each trial, the voltage data from the voltage amplifier was plotted against the average power of the laser. A slope was calculated per graph using a Matlab program. The table lists the slopes for all trials.

The plots generated by Matlab for the continuous wave data can be seen in Figure 2.24 and only the plot of trial 2 is presented in this section. Matlab plots for other trials of 1PA process can be located in the Appendix A.

As can be from the graph generated for 1PA process, the units of the data are given as volts. This is due to the usage of a voltage amplifier. The units for incident power as well as the GaP signal are converted to the
appropriate units using the gain factor in later calculations. The graph for 1PA process seen in Fig. 2.24 is a linear graph with well-behaved residuals. The data points seem to follow a linear path as anticipated by Equation 1.8 or the “1PA equation.”

The four separate slopes were averaged with their respective standard deviation. The average slope was used in the "1PA equation" in order to calculated the value of $z$. This value of $z$ is necessary for the calculation of the $\beta$ coefficient which will be covered in later sections. The average of the experimental slopes was calculated using weighted arithmetic mean which is an averaged value that is obtained by weighing each data point according to the significance of its contribution. The significance of the data point's contribution is dependent on its standard deviation. The equation that is used to calculate the weighted arithmetic mean is given below in Equation 2.1,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i (\sigma_i)^{-2}}{\sum_{i=1}^{n} (\sigma_i)^{-2}}.$$  \hspace{1cm} (2.1)

In Equation 2.1, the arithmetic weighted mean is given by $\bar{x}$ where the experimental slope of one of the trials is given by $x_i$ and $\sigma_i$ is its respective standard deviation. These experimental values are then added using Equation 2.1, giving the following

$$m_{\text{avg}} = \frac{2.079 (0.005)^{-2} + 2.080 (0.004)^{-2} + 2.062 (0.003)^{-2} + 2.063 (0.003)^{-2}}{(0.005)^{-2} + (0.004)^{-2} + (0.003)^{-2} + (0.003)^{-2}} = 2.068 \frac{V}{V}.$$

The standard deviation is given by $\sigma_i$ for the value of one trial. Therefore, the standard deviation for the weighted arithmetic mean is given by $\sigma_{\bar{x}}$ and the equation for calculating such an entity is is given as

$$\left(\sigma_{\bar{x}}\right)^2 = \frac{1}{\sum_{i=1}^{n} (\sigma_i)^{-2}}.$$  \hspace{1cm} (2.2)

If the values from the data are inserted into the equation, we get,

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{(0.005)^{-2} + (0.004)^{-2} + (0.003)^{-2} + (0.003)^{-2}}} = .002.$$

The average slope is calculated to be $m = 2.068 \pm .002$. Since this slope is directly calculated from the plot provided by Matlab, the units of $m$ are volts per volts. The units of $m$ must be converted into appropriate units
in order to be compatible with the “1PA equation,” and those units consist of amps per watts. This conversion of units can be obtained through the use of the gain factor from the relationship of \( i = Gv \) given to us by the voltage amplifier. For the continuous wave data, the value of \( G \) is 10 \( \mu \)A/V. The conversion of units for our average slope is

\[
m_{\text{avg}} = 2.068 \, \text{V} \left( \frac{1 \, \text{V}}{100 \, \text{mW}} \right) \left( \frac{1000 \, \text{mW}}{1 \, \text{W}} \right) = 20.68 \, \text{V/W}
\]

and

\[
m_{\text{avg}} G = \left( 20.68 \, \frac{\text{V}}{\text{W}} \right) \left( \frac{10 \, \mu \text{A}}{1 \, \text{V}} \right) \left( \frac{1 \, \text{A}}{10^6 \, \mu \text{A}} \right) = 0.000207 \, \frac{\text{A}}{\text{W}} = 2.07 \times 10^{-4} \, \frac{\text{A}}{\text{W}}.
\]

With our average slope being known and the value for \( \alpha \) provided by Yee and Chau at \( \alpha = 0.54/\text{cm} \), the value for \( z \) can now be calculated using the "1PA equation." The wavelength of the laser is given at 800 nm, and the value for \( hc \) is 1.239 eV\cdot\mu\text{m} so that the value of \( z \) is:

\[
m_{\text{avg}} G = \left( \frac{\lambda}{hc} \right) \alpha z
\]

Once the data is inserted into the equation, the value for \( z \) is

\[
\left( 2.07 \times 10^{-4} \, \frac{\text{A}}{\text{W}} \right) = \left( \frac{8 \times 10^{-7} \, \text{m}}{1.239 \times 10^{-6} \, \text{eV}\cdot\mu\text{m}} \right) \left( 54 \, \text{m}^{-1} \right) z.
\]

The numerical value for \( z \) is established to be 5.94 x 10\(^{-6}\) e\cdot\text{m} or to be more precise 5.94 x 10\(^{-6}\) give or take .002. The units for the value \( z \) are derived below,

\[
[z] = \left[ \frac{hc \, m}{\lambda \alpha} \right] = \frac{(\text{eV}\cdot\mu\text{m}) \, \lambda}{\text{(m)} \, \mu\text{A}} = \text{eV} \cdot \text{m} \, \frac{\lambda}{\text{V} \cdot \mu\text{A}} = \text{eV} \cdot \text{m} \, \frac{1}{\text{V}} = \text{e} \cdot \text{m}.
\]

Knowing the numerical value for \( z \) provides us with the opportunity to now calculate the \( \beta \) coefficient for the experimental data. In the next section, the experimental value for \( \beta \) will be calculated and compared to the values for \( \beta \) from previous publications.

### 2.3.2 Calculating the \( \beta \) of the GaP Crystal using Data from Double Photon Absorption

The numerical value for \( z \) was calculated in the previous section, and we can utilize it to calculate the \( \beta \) coefficient. The equation that mathematically represents two-photon absorption in the photodiode is given as Equation 1.11. For simplicity, it is reproduced here as
\[ m_{\text{avg}} = \left( \frac{1}{f \Delta t} \right)^2 \frac{\lambda \beta z}{2(\ln c) \pi (w_o)^2} \]

For the above equation, the bulk of the constants are already known due to the understanding of the instruments being utilized. The laser repetition frequency, \( f \), and the pulse width, \( \Delta t \), have values of \( 10^8 \) Hz and \( 10^{-13} \) s, respectively. With \( z \) already determined, all the variables are known within the "2PA equation" except beam spot size, \( w_o \), and the average slope, \( m_{\text{avg}} \). The knowledge that the focal length of the laser is around 100 mm and the beam diameter around 1 mm can be used to calculate the beam spot size. For \( w_o \), we get

\[
w_o = \frac{2 \lambda \beta}{\pi D} = \frac{2(8 \times 10^{-7} \text{m})(0.1 \text{m})}{\pi (0.001 \text{m})} = 5.09 \times 10^{-5} \text{ m}
\]

With beam spot size being \( w_o = 5.09 \times 10^{-5} \text{ m} \), all that is left is to find is the average slope for the data collected when the laser is in pulsed mode which is represented by the symbol \( m_{\text{avg}} \). The slope was obtained in the similar manner as for the single-photon absorption. For four separate trials, data was measured for the average power, \( P_o \), and for the voltage, \( v \), which corresponds to the signal from the photodiode. A plot of \( v \) vs \( (P_o)^2 \) was produced, and its slope was gathered from the Matlab program. The figure below shows the different slopes for the different trials of pulsed data.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Slope ((V/V^2))</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.362</td>
<td>.010</td>
</tr>
<tr>
<td>5</td>
<td>2.365</td>
<td>.010</td>
</tr>
<tr>
<td>6</td>
<td>2.351</td>
<td>.009</td>
</tr>
<tr>
<td>7</td>
<td>2.325</td>
<td>.009</td>
</tr>
</tbody>
</table>

**Table 2.8.** Table of the slopes obtained from the pulsed wave data for two-photon absorption in GaP material in the photodiode. The trials resulted in four different sets of data. From each trial, the voltage data from the amplifier was plotted against the squared average power of the laser. A slope was calculated per graph using a Matlab program. The table lists the slopes for the four trials.

The data plots generated by Matlab can be seen in Fig. 2.25 which is for the pulsed wave data of trial 1. For the Matlab plots of the other trials, please see Appendix B. It can be seen from Fig. 2.25 that the data does not
follow a complete linear path. The data points dip slightly downwards at higher incident power. The cause of this behavior in the plot can be due to two potential factors. One factor is saturable absorption which is a phenomenon where absorption of the material decreases at higher intensities such as 2PA. Saturable absorption works directly against 2PA since it slows down absorption. Another potential cause of the lack of linearity in the graph might be due to the Taylor Expansion used in the theoretical model. Taylor Expansion may be more accurate at lower intensity and less accurate at higher intensity which is why the data for 2PA is not as linear as data for 1PA. Furthermore, the Taylor Expansion was truncated at the first term, and future works on this thesis might examine the contributions of second or third orders to the accuracy of the model.

The average of the slope can be calculated for the pulsed data, and converted to the appropriate units using a gain factor as done in the preceding section. Below is the result for the average slope. Using Equation 2.1, we get

\[
\bar{\lambda} = \frac{\sum_{i=1}^{n} x_i (\sigma_i)^{-2}}{\sum_{i=1}^{n} (\sigma_i)^{-2}}
\]
and

\[ m_{\text{avg}} = \frac{2.362 (.010)^2 + 2.365 (.010)^2 + 2.351 (.009)^2 + 2.325 (.009)^2}{(.010)^2 + (.010)^2 + (.009)^2 + (.009)^2} = 2.350 \frac{V}{V^2} \]

The standard deviation is calculated in the same manner as the previous section. Equation 2.2 is used, and we get

\[ \left(\sigma_x\right)^2 = \frac{1}{\sum_i\left(\sigma_i\right)^2} \]

and

\[ \sigma_x = \sqrt{\frac{1}{(.010)^2 + (.010)^2 + (.009)^2 + (.009)^2}} = .005 \]

The average slope is given as 2.350 ± .005. In order for the average slope to be used, it must be converted into the proper units by a gain factor provided by the amplifier. The gain factor from the voltage amplifier is 2 \( \mu \)A for every volt for the pulsed data. The gain factor for the pulsed data is different than the gain factor for the continuous wave since the two-photon absorption generates larger amounts of photocurrent. Due to this, the signal from channel one requires less amplification. The gain factor is employed below in converting the units of the average slope from volts/volts to amps/watts.

\[ m_{\text{avg}} = (2.350 \frac{V}{V^2}) \left( \frac{1 V}{100 \text{ mW}} \right)^2 \left( \frac{1000 \text{ mW}}{1 \text{ W}} \right)^2 = 235 \frac{V}{W^2} \]

and

\[ m_{\text{avg}} G = (235 \frac{V}{W^2}) \left( \frac{2 \mu \text{A}}{1 \text{ V}} \right) \left( \frac{1 \text{A}}{10^6 \mu \text{A}} \right) = 4.7 \times 10^{-4} \frac{\text{A}}{\text{W}^2} \]

Lastly, the average slope is inserted into the “2PA equation” or Equation 1.11, allowing for the \( \beta \) coefficient to finally be calculated using experimental data. The calculations for \( \beta \) are

\[ m_{\text{avg}} G = \left( \frac{1}{f \Delta t} \right)^2 \frac{\lambda \beta z}{2 \left( \Delta c \right) \pi (w_o)^2} \]

and

\[ \left(4.7 \times 10^{-4} \frac{\text{Amps}}{\text{Watts}^2}\right) = \left( \frac{1}{(10^9 \text{ Hz}) (10^{-15} \text{ s})^2} \right) \frac{(8 \times 10^{-17} \text{ m}) \beta (5.94 \times 10^{-6} \text{ e-m})}{2(1.24 \times 10^{-6} \text{ eV-m}) \pi (5.09 \times 10^{-3} \text{ m})^2} \]

and

\[ \beta = 1.99 \times 10^{-16} \text{ m/W} = 1.99 \times 10^{-8} \text{ cm/MW} \]

The numerical value for \( \beta \) is 2.00 \( \times 10^{-16} \text{ m/W} \) or 2.00 \( \times 10^{-8} \text{ cm/MW} \) with a standard deviation of .007.
The numerical value for $b$ is $2.00 \times 10^{-16}$ m/W or $2.00 \times 10^{-8}$ cm/MW with a standard deviation of .007.

The units for $b$ are derived as

$$[b] = \left[ \frac{(m_e g) f^2 (\Delta f)^2 (hc)}{\lambda z} \right] = \left[ \frac{(\frac{\lambda}{w}) (Hz)^2 (s)^2 (eV m)}{(m)(e-m)} \right] = \left[ \frac{(\frac{\lambda}{w}) (eV m)}{e} \right] = \left( \frac{1}{V(A)} \right) (m) = \frac{m}{W}.$$

Unfortunately, the experimental value for our $b$ does not match values obtained for $b$ by other publications. The findings of Yee and Chau give the $b$ coefficient as $1.7 \times 10^{-3}$ cm/MW for the experimental and $0.19 \times 10^{-3}$ cm/MW for the theoretical. Other publications seem to match Yee and Chau in their findings for $b$. Bechtel and Smith presented $b$ at $2 \times 10^{-4}$ cm/MW for GaP crystal, seeming to be very close to Yee and Chau.

The table below provides a summary.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\delta$ (cm/MW)</th>
<th>Ref.</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical</td>
<td>$1.7 \times 10^{-5}$</td>
<td>Yee, Chau, 1974</td>
<td>1.06 $\mu$m, 50ns</td>
</tr>
<tr>
<td>Theoretical</td>
<td>$0.19 \times 10^{-3}$</td>
<td>Yee, Chau, 1974</td>
<td>1.06 $\mu$m, 50ns</td>
</tr>
<tr>
<td>Optical</td>
<td>$2 \times 10^{-4}$</td>
<td>Bechtel, Smith, 1976</td>
<td>1.064 $\mu$m, 1ps</td>
</tr>
<tr>
<td>Electrical</td>
<td>$2 \times 10^{-6}$</td>
<td>Present Work, 2016</td>
<td>0.800 $\mu$m, 100 fs</td>
</tr>
</tbody>
</table>

Table 2.10. Experimental and theoretical values for the two-photon absorption coefficient, $b$, for GaP material at room temperature. Table provides different values for $b$ obtained by other publications, as well as the value for $b$ obtained by the present work. With only a slight difference in wavelengths, it can be seen that the present work is off by several orders of magnitude when compared to values of $b$ from other publications.

As can be seen, the $b$ of our work is several orders off from the $b$ of other publications. There could be several factors contributing to the misleading results. One of the primary reason could potentially be due to saturable absorption which is often found in the presence of lasers at high intensity as in 2PA. Saturable absorption works against a 2PA phenomenon occurring in a material since saturable absorption decreases the ability of the material to absorb at high intensities. During saturable absorption, the electrons are all excited in the material and the material becomes very transparent. The output intensity of the laser beam as it exits the material increases. A higher and higher value for output intensity will be measured as the material becomes more transparent in the presence of saturable absorption which is how the phenomenon can be detected. The present work did not examine this, however, saturable absorption can be dealt with if the thickness of the GaP crystal is increased, allowing more material to interact with the high intensity laser. This might be a potential
area of interest to examine in the future in order to attain better results for $\beta$.

Other factors that might contribute to the difference in $\beta$ between present work and other publications might be due to the methods utilized. As can be seen in Table 2.10, the methods were optical in nature for the publications used for comparison. The optical method utilized was to analyze the intensity that exited the GaP crystal, meanwhile, the present work focused on analyzing the current generated inside the GaP photodiode. It is possible that the electrons produced in a semiconductor and measured as a current by a detector might behave differently or be harder to detect. Electrons produced by 2PA tend to be high-energy electrons that might not be picked up by the detector due to their high kinetic energy that might caused them to collide or be in rapid motion. This might also explain why a plot of a 1PA data in Fig. 2.24 produced cleaner results than a plot of 2PA data seen in Fig. 2.25 since electrons produced by 1PA tend to be of lower energy with easier detection by the detector.

Another potential cause of error that was not considered in the model is the dependence on intensity of the index of refraction. Index of refraction is dependent on temperature and intensity which may have contributed to potential errors in the calculation of $\beta$ in our work. The future work of this thesis could expand the theoretical model in order to included the effects of saturable absorption or varying index of refraction which are currently absent from the model. The model can be improved if these limitations are addressed.

### 2.3.3 Data Analysis of Scaling of 1PA and 2PA Processes

The theory for scaling was presented in Section 1.3.6. Here, the data obtained for the scaling processes of 1PA and 2PA is given which allows for the confirmation of 1PA and 2PA phenomenon inside the photodiode. Table 2.9 provides the slopes for the scaling of 1PA when a Matlab plot of $\ln(v)$ vs. $\ln(P_0)$ is generated. As stated in Section 1.3.6, the theoretical value predicted by the model presents a value of one for the slope of the $\ln(v)$ vs. $\ln(P_0)$ plot. If 1PA process does occur inside the material, then the theoretical slope should match the experimental slope.
Table 2.12. Slopes generated by the continuous wave data of 1PA process using scaling. The slopes are given by Matlab plots of ln(v) vs. ln($P_o$). As can be seen, the experimental slopes do not completely match the predicted value of $m = 1$ of the theoretical model.

The average slope and its standard deviation can be calculated in the same manner as prior slopes. With weighted arithmetic mean, the average slope, $m_{avg}$, is obtained for the scaling of 1PA process. The value that is generated is

$$m_{avg} = \frac{1.223 \cdot (.005)^2 + 1.241 \cdot (.005)^2 + 1.240 \cdot (.005)^2 + 1.240 \cdot (.005)^2}{(.005)^2 + (.005)^2 + (.005)^2 + (.005)^2} = 1.236 \frac{V}{V}.$$  

The standard deviation is also calculated in the same manner as in prior sections. For the standard deviation of the average slope of 1PA scaling, we obtain

$$\sigma_x = \sqrt{\frac{1}{(.005)^2 + (.005)^2 + (.005)^2 + (.005)^2}} = .003.$$  

The average slope for 1PA scaling is $1.236 \pm .003$. The experimental slope is slightly different than the predicted slope of one from the theoretical model of Section 1.3.6. Fig. 2.26 presents a Matlab plot of the scaling for a 1PA process of trial 2. For the Matlab plots of other trials, please see the Appendix C. It can be seen from graph of ln(v) vs. ln($P_o$) that the stronger the incident power of the laser the more well-behaved the GaP signal data. The residuals for this set of data varies a lot more in the region of lower incident power than in those regions of higher incident power. From Fig. 2.26, it can be seen that the residuals vary greatly in the region of lower power and there seems to be a gap of missing data around -2.4 V. This behavior at lower powers is most likely caused by the equipment being used in the data collection that generated noise within those range of incident power.
The data concerning the 2PA scaling was calculated with the same method as 1PA scaling, and the slopes for each of the trials with their respective standards of deviation is given in Table 2.10. The experimental slopes of the pulsed data for all of the trials did not completely match the predicted value of $m = 2$ from the theoretical model. The potential reason for the lack of consistency between the theoretical slope and the experimental slope could be due to the laser beam interacting with a foreign material before it reaches the GaP crystal. The GaP photodiode was not disassembled into its components for analysis in our present work. It is not known what is inside the photodiode since it was not taken apart. There is a possibility that the GaP crystal inside the photodiode has a material in front of it that is transparent to blue light and is holding the crystal in place. This foreign material could be interacting with the incoming laser beam and causing the data points to be skewed. For future works regarding this thesis, it would be beneficial to disassemble the GaP photodiode to see the structure within or do x-ray diffraction in order to get better understanding of the foreign material. Moreover, the presence of the foreign material inside the photodiode would also explain the $\beta$ coefficient of the present work not matching the $\beta$ of past publications as given in Table 2.10. The optical method employed by Yee and Bechtel only had a crystal in which the laser beam interacted. However, if our photodiode does contain foreign material, then the GaP crystal in the photodiode would be sandwiched between different materials that held it in place, forcing the
laser beam to interact with different objects. All of these factors could contribute to the 2PA and 1PA scaling have an experimental slope that does not match the theoretical slope predicted.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Slope (V/V)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.672</td>
<td>.008</td>
</tr>
<tr>
<td>2</td>
<td>1.659</td>
<td>.008</td>
</tr>
<tr>
<td>3</td>
<td>1.675</td>
<td>.008</td>
</tr>
<tr>
<td>4</td>
<td>1.680</td>
<td>.008</td>
</tr>
</tbody>
</table>

Table 2.13. Slopes for 2PA data using pulsed wave. The slopes are given by Matlab plots of ln(v) vs. ln(P_o). As can be seen, the experimental slopes do not completely match the predicted value of \( m = 2 \) of the theoretical model.

The calculations for the average slope and the standard deviation for the scaling of 2PA process is given. The values are \( m = 1.672 \pm .004 \) which are calculated as

\[
m_{\text{avg}} = \frac{1.672 (.008)^{-2} + 1.659 (.008)^{-2} + 1.675 (.008)^{-2} + 1.680 (.008)^{-2}}{4 (.008)^{-2} + (.008)^{-2} + (.008)^{-2} + (.008)^{-2}} = 1.672 \frac{V}{V}
\]

and

\[
\sigma_x = \sqrt{\frac{1}{4 (.008)^{-2} + (.008)^{-2} + (.008)^{-2} + (.008)^{-2}}} = .004 .
\]

Figure 2.27 presents a Matlab plot of the scaling for a 2PA process of trial 1. For the plots of other trials involving pulsed wave data, please see Appendix D. The 2PA scaling produces a graph whose residuals also vary greatly in the region of smaller incident power just like the results of 1PA scaling. The residuals, once again, vary in the regions of lower incident power and an empty gap of data is present around -2.4 V.
As stated before, the experimental values of the present work did not match the theoretical predictions of our work nor the numerical values obtained by other publications. The work of Chong, Watson, and Festy presented the slope of a log-log plot to be ~2 involving a 2PA process in a GaP crystal for a laser of 850 nm and ~140 fs. Our work should have produced a result similar to their publications since their method of analyzing 2PA was electrical in nature such as ours. Unfortunately, our work regarding the scaling of 2PA using log-log plots did not generate a slope of 2 like the work of Chong, Watson, and Festy. Our present work did not match this value nor our predicted value in the theoretical model. As previously stated, a potential explanation for this occurrence might be due to the construction of the photodiode. Since the GaP crystal is thin and delicate, it is possible that the GaP crystal is held in place by other material. The presence of this foreign material is likely interfering with the process and causing results to become slightly skewed. This would cause the slopes in the scaling as well as the $\beta$ coefficient to be a different value than anticipated.

2.4 Conclusion

A theoretical model was developed that provided a set of equations which were utilized in calculating $\beta$ coefficient from continuous and pulsed wave data. Along with attempting to obtain $\beta$, log-log plots were also
A theoretical model was developed that provided a set of equations which were utilized in calculating the coefficient from continuous and pulsed wave data. Along with attempting to obtain the coefficient, log-log plots were also employed in order to verify that 1PA and 2PA processes were occurring inside the GaP photodiode. The speculation for the lack of agreement within the results can be attributed to our theoretical model being incomplete. The theoretical model through which the data was analyzed did not account for factors such as saturable absorption or the limitations of the use of Taylor Expansion. Furthermore, the theoretical model treated 1PA and 2PA processes as if independent of each other. The two processes occur simultaneously inside the GaP material with one process dominating over the other. This simultaneous behavior is not accounted for in the theoretical model; therefore, it is difficult to ascertain how much of an impact 1PA occurrence has on the data of 2PA and vice versa. Future works regarding this thesis will have to take into account all of the factors that led to an incomplete theoretical model in order to obtain a complete one.

The methodology provided within this work may be improved on by obtaining a better understanding of the structure of the GaP photodiode to see if its construction plays a role in the skewed results, as well as evaluating how much impact intensity dependent index of refraction had on the results. If all of the factors and limitations are taken into account, a better model can be generated from the improvements of the current theoretical model in addition to devising a better methodological approach to data collecting.
For the 1PA process explained in Section 2.3.1, the GaP Signal vs Incident Power plots generated by Matlab are given. The three plots represent trials 3, 5, 6 in that order which were not presented in the section.
For the 2PA process of pulsed wave data presented in Section 2.3.2, the plots of GaP Signal vs Incident Power generated by Matlab are given. The plots are presented in order of trials 2, 3, and 4.
For the 1PA scaling process of Section 2.3.3, the log-log plots generated by Matlab are given for GaP Signal vs Incident Power. The plots are from trials 3, 5, and 6 in order.
For the 2PA scaling process of Section 2.3.3, the log-log plots generated by Matlab for GaP Signal vs Incident Power are given. The trials are 2, 3, 4, and are present in order.
Reference


