Hierarchical Structure Formation in the SDSS eBOSS Ly-alpha Forests

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Hierarchical Structure Formation in the SDSS eBOSS Lyman-alpha Forests

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ABSTRACT In this study, we examine hierarchical structure at large redshifts utilizing Sloan Digital Sky Survey (SDSS) Lyman-alpha forest data. Lyman-alpha forests are absorption lines in the spectrum of quasars that serve as tracers for clouds of primordial hydrogen. These data serve as a 1-dimensional probe of the matter density field at high redshift. Using a measure sensitive to hierarchical structure formation assembled around a discrete wavelet transform, we were able to detect hierarchical structure formation in the spectrum of every quasar studied. Some systems showed power law behavior in the scale-scale correlation, while others did not have a preferred scale. Future work exploring the entirety of the data set is needed to determine what, if any, patterns emerge, as well as the underlying factors involved with potential patterns. Along with a deeper analysis of the data set, we hope to investigate factors such as redshift dependence with extended research.

INTRODUCTION
In the standard concordance model of the universe, structure forms hierarchically when small densities in the distribution of dark matter are gravitationally attracted to each other to form larger structures. These structures form what is known as the cosmic web because matter is distributed across the universe in filaments and clusters that give web like appearance. While this model of structure formation has been generally confirmed for recent times, the contemporary advent of large and deep galaxy surveys makes it possible to not only confirm this model for earlier epochs but may also allow for the understanding of the evolution of hierarchical structure formation.

With this study, we seek to understand the evolution of large-scale structures based on Lyman-alpha absorption data from quasars as proxies for dark matter and its formation. A quasar is a young galaxy, which is particularly bright (on average, quasars are 10-100,000 times brighter than the Milky Way [Redd, 2018]). Since light from quasars is so intense, quasars can be observed at extremely large distances from the Earth. In fact, the quasars observed in this study
are billions of light years away. While light traverses these large distances from quasars to Earth, it sometimes encounters large hydrogen clouds. These clouds are presumed to be captured by underlying dark matter and thus serve as a probe of the distribution of dark matter along the line of sight between the Earth and quasar. The interaction between the quasar's energetic light and the hydrogen clouds is detected in spectral analysis as absorption lines called Lyman-alpha lines, with a group of Lyman-alpha lines being a Lyman-alpha forest. The amount of absorption is a measure of the amount of hydrogen, and therefore, dark matter in any particular location. By examining Lyman-alpha forests, we can trace the evolution of large-scale structures over time.

We are specifically interested in detecting the hierarchical formation of large-scale structure using the Sloan Digital Sky Surveys (SDSS) Baryonic Oscillation Spectroscopic Survey (BOSS) (Dawson et al: 2018). As part of this survey, SDSS conducted measurements of Lyman-alpha forests at high-redshifts \( z > 2 \). This survey provides an ideal data set with which to study the hierarchical formation of structure as it is both deep (that is, at large distances) and wide (as it covers most of the northern sky). To detect the hierarchical structure formation, we make novel use of wavelets which allow us to probe the correlation in structures at multiple scales.

With this analysis, we were able to determine that there is hierarchical structure in quasar's spectra of the SDSS BOSS survey. Regarding the nature of this hierarchy, we established it to be one of two forms: hierarchy exhibiting a power law distribution, and hierarchy that did not have a favored scale.

**THEORY**

Once light leaves quasars, it passes through the Intergalactic Medium (IGM) (Shu, 2009). The IGM is what fills the space between objects in the universe such as galaxies and is made up of diffuse gas and dust, consisting mainly of neutral hydrogen gas and ionized hydrogen gas. The basic idea for the existence of hydrogen clouds is that as one goes back in time, there should be an increasing amount of baryonic matter in the form of gas. The reason for this is that gravitational attraction has not had enough time to draw this gas into the denser regions of the universe. As energetic light from strong sources like quasars passes through hydrogen clouds, there is a high probability these clouds will absorb the light. In physics, absorption of electromagnetic radiation describes how matter takes up photon energy and transforms this energy. Additionally, the probability of absorption should increase the further one goes back in time \( (i.e.: \) at larger distances from Earth), due to the higher abundance of gas. In fact, the probability of light being absorbed is so high, that even if only a small fraction of the mass in the universe were in the form of neutral hydrogen, all the light from this part of the spectrum would be absorbed. This turns out not to be observed because much of the hydrogen is ionized, however, enough of the hydrogen is in its neutral state that these clouds serve as excellent tracers of the matter distribution along the line of sight.

![Figure 1](image)

**Figure 1.** In both graphs, the horizontal axis depicts emitted wavelength in angstroms, and the vertical axis depicts flux. The top figure is a quasar spectrum with minimal Lymanalpha absorption, since the quasar itself is relatively close to Earth and there are likely less hydrogen clouds between the quasar and Earth. The bottom figure is a quasar spectrum with extensive Lyman-alpha absorption present (and a clear Lymanalpha forest). Since the quasar itself is more distant, there are more hydrogen clouds in its path. Both figures are taken from Wright (Wright, 2004).

In Figure 1, we show an absorption spectrum of a nearby quasar and a distant quasar (Wright, 2004). The horizontal axis corresponds to emitted wavelength in angstroms and the vertical axis
corresponds to transmitted flux (in our case, the flux is the amount of light that passes through the detector used by SDSS). Note that the wavelength given on the horizontal axis is emitted wavelength and not observed wavelength (if this were not the case, the nearby quasar and distant quasar would be at very different horizontal positions due to redshift). All other figures pertaining to this research depict observed measurements. Dips in the graph indicate a region between the quasar and Earth where light was absorbed, presumably by a neutral hydrogen cloud. As one can see, the distant quasar's spectrum has more dips than that of the nearby quasar, which indicates more absorption along the line of sight. This is due to the higher probability of absorption farther from Earth, reflecting the more plentiful amount of diffuse hydrogen gas further back in time.

Neutral hydrogen absorbs light of wavelength 1216 angstroms, and we can use this fact to figure out how far away specific areas of absorption are from Earth as a function of redshift. When we say redshift, which we denote $z$, we are describing the shift in wavelength observed as objects move farther from Earth. As a body moves away from an observer, the light it emits is stretched to a longer wavelength than its original emitted wavelength. Just like with the Doppler Effect and sound waves, as a wave source (which in the case of this research is a quasar, and in the case of the Doppler Effect is a sound source) moves away from an observer, the observer records a wavelength that is longer than the wavelength observed at the source. When we consider the visible spectrum, the largest wavelengths pertain to the color red, so it is said that a lengthened wavelength has been shifted towards the red end of the spectrum. Hence, when light shifts towards the red, longer-wavelength side of the spectrum as an object moves away, its light is said to be redshifted. The most distant objects in the universe are the most redshifted, while closer objects are less redshifted.

In Figure 2, we show the basic physics of how the absorption characteristic of the Lyman-alpha forest works. Quasars emit a wide spectrum of light, including light in the right range to excite the ground state electron of neutral hydrogen to its first excited state. As the radiation from the quasar interacts with neutral hydrogen clouds, the quasar's radiation in this wavelength will be absorbed. Thus, the quasar's spectrum will be missing this contribution. Historically this part of the hydrogen spectrum is called the Lyman-alpha spectrum. The electron will quickly fall back to its ground state by emitting a photon at the same wavelength that was absorbed. The emitted photons create a transmitted flux of radiation from the clouds. In practice, either the absorption spectrum or the transmitted flux can be used to probe the distribution of matter along the line of sight of the quasar. Clouds with high column densities shield themselves from the ionizing radiation. Typically, this happens at densities $N > 1019 \text{ cm}^{-2}$. These are called damped systems and do not form part of what are typically considered Lyman-alpha forests. Detailed modeling matched by observations has shown that the majority of Lyman-alpha absorbers are located in low-density columns formed by gravitational collapse of the IGM. It appears that the gas from Lyman-alpha forest interactions is the original source of matter from which galaxies form.

Figure 2. A depiction of hydrogen absorption at the atomic level. A light source (in the case of this research, a quasar) emits light at a wavelength of 1216 angstroms towards the neutral hydrogen atom. When hydrogen interacts with photons of this wavelength, it absorbs the light and uses this energy to move its electron from $n = 1$ (ground) to $n = 2$, resulting in an absorption line in its spectrum. A group of these absorption lines is known as a Lyman-alpha forest.

METHODS
The wavelet transform is a mathematical tool that helps break a signal or function into different scale components. In this study, the discrete
wavelet transform (DWT) is utilized to determine scale-scale correlations. A scale-scale correlation is used to determine the likelihood of matter clustering hierarchically. The DWT is computed by breaking the signal vector (which in the case of this research, is the binned vector of absorption fluxes taken from the quasar spectrum) into two vectors referred to as the app coefficient and the detail coefficient. The app coefficient represents a local average of the field density, and the detail coefficient represents local fluctuations from the local average. An example calculation of both of these coefficients is depicted in Figure 3, and Equation 1 is employed to carry out this calculation. The app coefficient is calculated by pairing off adjacent elements of the signal vector and computing their average. After all the local averages have been computed for the app coefficient, there are half as many coefficients as there were in the original signal vector. The detail coefficient is calculated similarly, since it is also created by pairing off adjacent elements of the signal vector. Instead of computing the average of the two elements, however, their difference is taken and divided by two. Like the app coefficient, after the local fluctuations are calculated for the detail coefficient, there are half as many coefficients as there were in the original signal vector. Once both the app coefficient and detail coefficient have been calculated, one pass of the wavelet has been computed. To go through another pass, the same process outlined above is completed on the app coefficient from the previous pass. Since the number of elements in the app and detail coefficients is halved with each pass, the number of possible passes depends on the length of the original source vector. The detail coefficients from the DWT are then substituted into a scale-scale correlation equation for further analysis. As described in Pando et al., the difference coefficient is calculated by way of the following equation (Pando et al., 1998):

$$\tilde{\delta}(x) = \sum_{j=0}^{J-1} \sum_{l=0}^{2^j-1} \tilde{\delta}_{j,l} \psi_{j,l}(x)$$

In the case of this research, $\delta(x)$ represents the difference coefficients procured by the DWT. Equation 1 is a general description that can be used for any decomposition, and $\psi_{j,l}(x)$ is a complete and orthogonal wavelet basis. Subscripts $j$ and $l$ correspond to scale and position within the vector, respectively. The basis $\psi_{j,l}(x)$ is orthogonal to both $j$ and $l$.

In Figure 3 we graphically map the process of the wavelet decomposition. A source vector, $s$, in this case consisting of 8 flux data points, is passed through two wavelet filters. One filter generates a local average, the app coefficient, which is depicted in the pink boxes on the left. A quick review shows that entries in the first app coefficient are the local pairwise averages of the original signal. The other filter generates average local fluctuations, the detail coefficient, which is shown in the blue boxes on the right. Hierarchical clustering is characterized by local relations between large-scale structures, therefore an accurate decomposition model should also be localized and orthogonal. The DWT is constructed on a localized, orthogonal basis, making it an ideal tool to compute scale-scale correlations.
The flux data we receive from SDSS are raw data, so SDSS has not made corrections to the data set. SDSS has supplied us with a variety of correction vectors we were able to manipulate to clean our flux data. First, we corrected for damping in our spectra and pipeline noise. Damping in the spectra refers to clouds that are too dense to be included in a Lyman-alpha forest, and external objects blocking out the quasar spectrum. On the contrary, when we discuss the pipeline, we are referring to the SDSS imaging pipeline, which processes raw data and compiles this information into a Flexible Image Transport System (FITS) file format. Therefore, corrections for pipeline noise address any contribution from signal processing that could skew the raw data. Correction vectors corresponding to damping and pipeline noise are multiplied and divided by the raw flux vector, respectively. Additionally, we divide out the quasar continuum in the rest frame 1040-1600 angstrom region to isolate Lyman-alpha forest transmission, and avoid any background noise from the source quasar. Further, in order to ensure the objects selected for our study were suitable for Lyman-alpha analysis, we filtered out fluxes that corresponded to redshifts below 2.15, and only used Lyman-alpha forest pixels in the wavelength range 1041-1185 angstroms.

We chose to work in comoving space as opposed to redshift space for the duration of this study. Comoving distances are preferred because they are less susceptible to redshift distortions. Comoving space is a system where distance is considered constant, because the expansion of space is factored out. The comoving distance between two points always refers to the proper distance at the current time (i.e. time = now). In order to determine the proper distance (i.e., the actual, physical distance between two objects) at a specific time, the constant comoving distance is multiplied by a scale factor relating to the specific time in question. At the current time, this scale factor is equivalent to 1, since the comoving distance is equivalent to the proper distance. If we want to look into the past, however, the scale factor is less than one, because the proper distance is less than the comoving distance due to the expansion of the universe. Since our spatial data from SDSS is given to us in redshift space, we need to convert these values to comoving coordinates. The relationship between redshift and distance is:

\[ D_c = D_H \int_0^z \frac{dz'}{E(z')} \]

where

\[ E(z) = \sqrt{\Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_{\Lambda}} \]

In these equations, \( D_c \) is the calculated comoving distance, \( z \) is the cosmological redshift of the object, \( D_H \) is the Hubble Distance and is calculated as follows: \( D_H = c / H_0 \) where \( H_0 \) is the Hubble parameter, \( c \) is the speed of light, and \( \Omega_i \) with \( i = m, k, \Lambda \) is the critical density of the mass, curvature, and dark energy respectively. For all parameters, we used the concordance values obtained by various methods. Specifically as described in Hogg (Hogg, 1999), the mass density, \( \rho \), of the universe and the value of the cosmological constant \( \Lambda \) can be made into dimensionless density parameters \( \Omega_m \) and \( \Omega_{\Lambda} \) by:

\[ \Omega_m = \frac{8\pi G\rho_0}{3H_0^2} \]

\[ \Omega_{\Lambda} = \frac{\Lambda c^2}{3H_0^2} \]

The quantities sub-scripted with 0s are to be evaluated at the present epoch. \( G \) is Newton’s Gravitational Constant. In our study, we completed these calculations and used \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0.7 \). Each parameter has units of \( h^{-1}M_\odot D^{-3}h \), where \( h \) is Planck’s Constant, and \( M_\odot \) is the mass of the Sun. The third density parameter, \( \Omega_k \), can be calculated with the following relation:

\[ \Omega_m + \Omega_{\Lambda} + \Omega_k = 1 \]

By this relationship, we use a \( \Omega_k \) value of 0. These parameters resolve the geometry of the universe if it is homogeneous, isotropic, and matter-dominated. With these equations, redshift is converted to comoving distance.
One of the most important facets of our data set is how far apart our flux readings are from one another. The DWT compares readings at different scales, and in order to accomplish this in relation to distance, the distance between data points must be taken into account within the flux vector. When a data point is binned within a vector, its location in the vector indicates its distance from other data points in the vector.

For this study, we decided to use a bin size of 0.6 Mpc, so each data point is 0.6 Mpc away from its adjacent data point. One Mpc denotes a megaparsec, with the pre_x mega equating to $10^6$ and a parsec equaling 3.26 light years of distance. Thus, a distance of one Mpc is equivalent to $3.26 \times 10^6$ lightyears. We chose a specific bin size of 0.6 Mpc to ensure that the minimum amount of data points ended up in the same location of the vector (because if two data points are less than 0.6 Mpc apart, they will be placed in the same bin, so one reading will be masked). Additionally, we needed to make sure our bin size was not too small. If this were the case, the data points would be too far away from one another within the vector (which would make the results of the DWT less meaningful). After working with the data set for an extended period, we decided a bin size of 0.6 Mpc would bin the data in the most efficient and advantageous way for our analysis.

We use the scale-scale correlation function first defined in Pando et al. (Pando et al., 1998):

$$C_{j+1, j}^{p,p} = \frac{2^{j+1} \sum_{l=0}^{2^j-1} \left( \frac{\delta_{j,l}^{p} \delta_{j+1, 2l}^{p}}{\sum_{l=0}^{2^j-1} \left( \delta_{j,l}^{p} \right) \sum_{l=0}^{2^{j+1}-1} \left( \delta_{j+1,l}^{p} \right)} \right)}{2^j \sum_{l=0}^{2^j-1} \left( \delta_{j,l}^{p} \right)}$$

As described in Pando et al., these scale-scale correlation measures are symmetric and normalized with even orders $p = q$. The $\delta$ quantities are the difference coefficients calculated with the DWT, and the subscripts $j$ and $l$ correspond to scale and position within the detail coefficient, respectively. Therefore the difference between $\delta_{j,l}$ and $\delta_{j+1,l}$ is that $\delta_{j,l}$ corresponds to one particular scale, or pass of the wavelet, and $\delta_{j+1,l}$ corresponds to the next pass of the wavelet. Due to normalization of the denominator, when randomized bootstrapped data are entered into Equation 7, $C_{j+1, j}^{p,p} = 1$ for $P \geq 2$, so there are no scale-scale correlations of order greater than 2. Equation 7 provides a sensitive quantification of correlations between structures located at adjacent scales. This equation also specifically distinguishes between random bootstrapped distributions and hierarchical clustering.

As previously mentioned, to glean a random set of data we used a method known as bootstrapping. When data are randomized with bootstrapping methods, actual flux data from SDSS is randomized independently of their analogous spatial coordinates. For this particular data set, each recorded flux has its own redshift coordinates. This redshift reading is converted to co-moving distance with Equations 2 and 3. When data are bootstrapped, fluxes are randomized separately from their comoving coordinates, so each flux is paired with a random comoving distance from the data set, as opposed to its recorded distance. This way we are able to create a set of random data that is not only indiscriminate, but contains points that are representative of the data set.

![Figure 4](https://via.library.depaul.edu/depaul-disc/vol8/iss1/2)
In Figure 4, we present an example of the bootstrapping method. The table on the left contains model flux data with their corresponding comoving coordinates. Each flux data point has its own associated comoving distance, and each of these flux-comoving pairs have their own unique color. The table on the right contains the same data as the table on the left, but in this table the data have been bootstrapped. The comoving coordinates have been shuffled independently of their partnering fluxes. With this method of randomization, we can test the behavior of random data in the scale-scale correlation equation (Equation 7), while still using values that are typical to the data set.

RESULTS AND DISCUSSION
Figure 5 depicts a typical result of the scale-scale correlation function plotted against scale \( j \). The horizontal axis is the scale \( j \) at which the flux readings are correlated, and the vertical axis is the correlation coefficient, \( C_{\rho'}^p \), derived using Equation 7. Since our flux readings were binned according to comoving distance, larger scale \( j \) values indicate larger distances. Therefore, the correlation value pertaining to a \( j \) value of 1, for example, indicates the correlation between hydrogen clouds that are only 0.6 Mpc apart. At each subsequent \( j \), the physical distance doubles, so \( j = 2 \) pertains to a comoving distance of 1.2 Mpc, \( j = 3 \) pertains to 2.4 Mpc, and so on. Thus, the higher the scale \( j \) value, the further apart two correlated flux readings are. The red points and error bars are generated from bootstrapped random data, so they all have correlation values at or near 1, and the black points are the real data from the SDSS Lyman-alpha transmitted flux.

All Lyman-alpha Forest data we studied showed a clear deviation from the bootstrapped model. This deviation from the random data indicates hierarchical structure formation. Some data, but not the data showed in Figure 5, seemed to show power law behavior in this measure, but others did not follow a power law trend.

We were expecting to see a power law relationship, specifically in the form of a decreasing exponential, due to gravitational effects. At smaller scales, objects are closer together, so the effects of gravity with regards to bringing hydrogen clouds together and/or splitting them up (i.e. hierarchical structure formation) are larger than gravity's effects on clouds that are farther away from one another. Thus, we would expect higher correlation values at small scales than at large scales because of gravity. As previously mentioned, some figures did display a decreasing power trend, however many others (such as Figure 5) did not. This does not take away from the fact that hierarchical structure formation was detected, it simply means that the relationship is more complicated than one that is only dependent on gravitational attraction. As of now we could not determine any common trend other than significantly different from random data, and we have not yet investigated these other factors governing hierarchical structure formation.

CONCLUSION
Using the Lyman-alpha forest samples from SDSS, we were able to develop a method of extracting hierarchical information regarding Lyman-alpha forests. We utilized a scale-scale correlation equation based in a wavelet transform as a tool to extract this information. After appropriately treating the data, we applied the scale-scale correlation to the corrected SDSS data. Our study clearly shows that Lyman-alpha forests are hierarchically clustered, since the
correlation parameters derived from real data do not match correlation parameters from random data. As of now, we have not been able to determine the nature of the hierarchy. The fact that the data do not show a clear trend indicates that there are other factors we need to take into account with further analysis. The hierarchy measured with this study is strictly dependent on gravitational attraction between neutral hydrogen clouds. If this was the only factor guiding hierarchical clustering, we would expect to see a power law distribution in our final figures. While some graphs exhibited this behavior, others did not have a preferred scale. This indicates that there are more components guiding structure formation to take into account. Some of these influences could include, but are not limited to, latent hydrogen cloud temperature, peculiar cloud velocities, and perhaps some sort of redshift dependence. We hope to investigate these factors with further research.

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