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Anomaly Discovery and Arbitrage Trading*

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Anomaly Discovery and Arbitrage Trading

Abstract

Our model of anomaly discovery has implications for both *asset prices* and *arbitrageurs' trading.* Consistent with existing evidence, the discovery of an anomaly reduces its magnitude. Our evidence based on 99 anomalies is consistent with new predictions that the discovery of an anomaly reduces the correlation between the returns its deciles 1 and 10, leading to diversification benefits for passive investors. These effects *become* linked to the aggregate trading of hedge funds only *after* discovery. Hedge funds increase (reverse) their positions in exploiting anomalies when their aggregate wealth increases (decreases), further suggesting that these discovery effects operate through arbitrage trading.

Keywords: Anomaly, Arbitrage, Discovery, Arbitrageur-based asset pricing.

JEL Classifications: G11, G23.

1 Introduction

A significant portion of the asset-pricing literature has been devoted to "anomalies," empirical patterns that appear inconsistent with existing benchmark models. One approach to interpreting anomalies is risk-based. Consider the value premium as an example. It is first documented by Basu (1983). Since then, numerous models have been proposed to explain why value stocks are riskier (than what CAPM implies) and so should have higher expected returns.

This approach abstracts away from the *discovery aspect*. That is, investors in those models know that value stocks are riskier and demand higher returns. As expected by those investors, higher average returns are realized for value stocks in the data. In this view, there is no real discovery: Professor Basu was the last one in the world to find out about the value premium. Investors knew about this return pattern all along. Essentially, in this view, the amount of capital that responds to Basu's findings is too small to meaningfully alter asset prices.

In contrast to the view above, it seems natural to expect discoveries to have significant effects on investors' decisions and asset prices, as discoveries in academia have had increasingly important influences on the asset management industry, especially since the rise of the hedge fund industry in the 1990s. Many prominent asset management companies regularly organize academic seminars and conferences. Some explicitly claim that they identify investment ideas from academic research.[1] Hence, in this paper, we focus on the discovery aspect by analyzing a stylized model of anomaly discovery and testing its new predictions empirically. The goal of the model is to have a simple framework to analyze the effects of anomaly discovery and provide guidance for designing our empirical tests.

We first consider a model of "risk-based" anomaly. There are two assets (asset 1 and asset 2) that have the same distribution of future cash flows. However, investors find asset 1 riskier because their endowment is correlated with asset 1's cash flow, but not with asset 2's. Consequently, in equilibrium, asset 1 has a lower price and a higher expected future return than asset 2. We call this return pattern "a risk-based anomaly" in the sense that it is caused by investors' risk consideration. When this anomaly is discovered, some agents, who we call "arbitrageurs," become aware of the return pattern. Importantly, arbitrageurs find the return pattern worth exploiting, perhaps because they have a different labor income profile and do not face the endowment risk that investors do.

[1] Take Dimensional Fund Advisors as an example. According to its website, as of June 30, 2014, it manages \$378 billion. Academic research appears to have a deep influence on its operation, as its website states: "Working closely with leading financial academics, we identify new ideas that may benefit investors."

To analyze the discovery effect, we construct an equilibrium without these arbitrageurs, which we call the "pre-discovery equilibrium," and an equilibrium with these arbitrageurs, which we call the "post-discovery equilibrium." The discovery effect is captured by the difference between the pre- and post-discovery equilibria.

The notion of "risk-based" in our stylized model is different from that in traditional models. But that is the point. Traditional models abstract away from discovery and all investors know the return patterns and do not respond to the "discovery of the anomaly." In our formulation, however, arbitrageurs *become aware* of the anomaly and find it worth exploiting. As pointed out in Cochrane (1999), this discovery aspect "is (so far) the least stressed in academic analysis. In my opinion, it may end up being the most important." Moreover, the traditional formulation can be viewed as a special case in our model, where no arbitrageurs find the anomaly worth exploiting.

Our model has two sets of implications on *asset prices*, as well as associated implications on *arbitrageurs' trading*. First, after the discovery of an anomaly, its return (i.e., the return from a long position in asset 1 and a short position in asset 2) decreases and becomes more correlated with the returns from other existing anomalies. The result that the discovery of an anomaly reduces its magnitude follows directly once we recognize that the discovery brings in arbitrageurs. Let us use the value premium as an example. It has been proposed that value stocks are riskier because they are more exposed to the business cycle. Arbitrageurs, however, may not be as concerned about this risk and choose to exploit this anomaly and consequently reduce its magnitude.[2] The correlation with other anomalies is due to a wealth effect when arbitrageurs exploit both existing anomalies and the newly discovered one. Suppose the return from existing anomalies is unexpectedly high one period, thus increasing arbitrageurs' wealth. Everything else being equal, arbitrageurs will allocate more investment to all their opportunities, including the new anomaly. This higher investment pushes up the price of asset 1 and pushes down the price of asset 2, leading to a high return from the new anomaly. Similarly, a low return from existing anomalies leads to a low return from the newly discovered one. Hence, the wealth effect increases the correlation between the new anomaly return and the returns from existing anomalies.

Second, the discovery reduces the correlation between the returns of assets 1 and 2, and this effect is stronger when arbitrageurs' wealth is more volatile. This is because arbitrageurs increase (reverse) their positions in exploiting the anomaly when their wealth increases (decreases).

[2] For example, in an article written by several senior managers at AQR, Asness, Frazzini, Israel, and Moskowitz (2014) state that "[w]e are fans of both momentum and value..." They also state that "none of this debate [about whether momentum is due to risk or mispricing] should diminish momentum as a valuable investment tool."

Specifically, after the discovery, arbitrageurs have a long-short position in assets 1 and 2, as well as investments in other opportunities. Suppose the arbitrageurs' wealth increases due to, say, a high return from their investments or fund flows from their investors. They will buy asset 1 and sell asset 2. This increases asset 1's return but decreases asset 2's. Similarly, when arbitrageurs' wealth decreases, they will unwind some of their long-short positions, i.e., sell asset 1 and buy asset 2, which decreases asset 1's return but increases asset 2's. In both cases, arbitrageurs' wealth shocks push the returns of the two assets to opposite directions, reducing their correlation. This intuition also suggests that the effect is stronger when arbitrageurs' wealth is more volatile. Since the correlation between assets 1 and 2 is reduced, a natural implication is that, ceteris paribus, the volatility of holding both assets 1 and 2, i.e., the market portfolio, is reduced, leading a diversification benefit for passive investors. This diversification benefit is stronger when arbitrageurs' wealth volatility is higher.

We also analyze a version of our model where the anomaly is due to "mispricing." Specifically, we modify the previous model so that investors do not have the hedging need in asset 1, but mistakenly believe that asset 1's future cash flow is lower than asset 2's. Our analysis shows that the discovery of this mispricing-based anomaly has the *same* two sets of predictions on asset prices, as well as arbitrageurs' trading activities. The first set of implications on prices are consistent with existing empirical evidence. For example, McLean and Pontiff (2016) analyze the post-discovery performance of 97 anomalies, and find that after the discovery of an anomaly, it returns decay by 58% on average, and become more correlated with the returns from existing anomalies.

The second set of implications on asset prices are new to the literature. We empirically examine them, as well as their implied *arbitrage trading activities*. Our tests are based on a comprehensive set of 99 anomalies, which are constructed using widely-accessible public data. We first test whether the correlation between deciles 1 (the long leg) and 10 (the short leg) of an anomaly decreases after the discovery of the anomaly. For each anomaly, we use a 5-year rolling window to estimate the correlation coefficient between the monthly excess returns of deciles 1 and 10 during 1963–2015. To control for its potential time trend, our analysis focuses on *excess correlation*: the correlation between deciles 1 and 10 minus the correlation between deciles 5 and 6. The idea is that arbitrageurs are likely to take larger long-short positions in deciles 1 and 10 than in deciles 5 and 6. Hence, the correlation between deciles 5 and 6 should have little discovery effect, but should share the common time trend with the correlation between deciles 1 and 10.

For each anomaly, we construct a dummy variable that takes the value of 0 before the "discovery" of the anomaly and 1 afterwards. We use the publication time of the paper that documented the anomaly (or latest working paper dates for unpublished papers) as a proxy for the discovery time. We pool the anomalies together, and run a panel regression of the excess correlation on the discovery dummy. The coefficient for the dummy variable is -0.05 ($t = -6.39$), implying that, on average, the discovery of an anomaly reduces the correlation by 5%.

Our evidence shows that there is a link between the excess correlation and the aggregate hedge funds' activity, further supporting the view that the discovery effect operates through arbitrage trading. Our model implies that after the discovery of an anomaly, the correlation between deciles 1 and 10 becomes *more* negatively correlated with the volatility of arbitrageurs' wealth.[3] Since we cannot directly measure the aggregate wealth of all arbitrageurs, we use the aggregate hedge fund asset under management as a proxy. We run a panel regression of the excess correlation on the interaction term of the discovery dummy and the hedge fund wealth volatility. Our model implies that the hedge fund wealth volatility reduces the correlation between deciles 1 and 10 *after* the discovery, and hence that the coefficient for the interaction term should be negative. This is indeed the case. The coefficient for the interaction term is -0.05, with a t-statistic of -2.37.

Our interpretation suggests that the discovery effect should be stronger if the discovery attracts more attention. To test this implication, we use the Google citation count of the original study that discoveries an anomaly as a proxy of the attention to the anomaly. Highly-cited anomalies by both academic and practitioner journals are likely to be the anomalies with persistent profitability and tradability, thus attracting more arbitrage trading. We run a citation-weighted least square regression, which assigns higher weight to anomalies with higher citation counts. Consistent with the implication, the above two effects become significantly stronger once we weight anomalies by their citation counts. The coefficient for the discovery dummy is 0.09, with a t-statistic of -9.11. The coefficient for the interaction term is -0.13, with a t-statistic of -3.00.

These post-discovery correlation changes are large. For example, the post-discovery correlation reduction represents 36% and 64% of the standard deviation of the correlation coefficient when anomalies are equal-weighted and citation-weighted, respectively.

To further assess the economic significance of this reduction in correlation, we examine its diversification benefit: the reduction of the correlation coefficient between deciles 1 and 10 reduces the volatility of holding the aggregate portfolio of deciles 1 and 10. To see this, we examine the

[3] This prediction is the opposite of the implication from the common intuition that arbitrageurs' wealth tends to be more volatile when the market is more volatile (e.g., in a financial crisis). Since stocks tend to be more correlated when the market is more volatile, this intuition implies that the correlation between deciles 1 and 10 should be *increasing* in the volatility of arbitrageurs' wealth.

volatility of the equal weighted portfolio deciles 1 and 10 in excess of the market volatility. We find that, on the equal (citation) weight basis, the discovery of an anomaly reduces this volatility by 21% (61%) of its pre-discovery level. Since the set of stocks in deciles 1 and 10 of an anomaly change over time, the diversification benefits are effectively shared by investors who hold the market portfolio. That is, arbitrageurs' trading provides a diversification benefit for passive investors who hold the market portfolio. Our evidence further shows that this effect is stronger when arbitrageurs' wealth volatility is higher.

Finally, we directly examine the two model implications on arbitrage trading. First, we examine whether hedge funds trade *more* in the direction of exploiting an anomaly (i.e., buy decile 1 and sell decile 10) *after* its discovery. Second, we test whether, after the discovery of an anomaly, hedge funds increase their positions in exploiting the anomaly when their wealth increases, and reverses their positions when their wealth decreases.

We construct two alternative arbitrage trading measures. Our main measure is based on the changes in the aggregate hedge fund holdings. We identify hedge funds in the 13F institutional holdings filings. For each anomaly, we compute the aggregate hedge fund position changes in decile 1 and in decile 10. We use the difference between the two position changes as a proxy for arbitrageurs' trading on the anomaly. A positive (negative) value means that hedge funds are trading in the "right" ("wrong") direction of exploiting the anomaly. The other measure is similarly defined based on the difference between the short interest changes of the two deciles.

Consistent with our model predictions, the regression results based on both measures suggest that an anomaly discovery leads to an increase in arbitrage trading in the direction of exploiting the anomaly. The magnitude of such increase is between 2% and 7% of the total shares outstanding of the average stocks traded in the two deciles. Moreover, the aggregate arbitrage positions increase (decrease) when the aggregate asset under management of hedge funds increase (decrease). A one standard deviation post-discovery increase (decrease) in the aggregate asset under management by hedge funds leads to an increase (decrease) in arbitrage trading that exploit the anomaly by between 2% and 7% of the total shares outstanding. As expected, the results are stronger when anomalies are citation-weighted in the estimation.

On the theoretical front, our paper is closely related to the analyses of arbitrageurs' risk-bearing capacity (e.g., Dow and Gorton (1994), Shleifer and Vishny (1997), Xiong (2001), and Kyle and Xiong (2001)). More broadly, our paper belongs to the literature that explores the role of arbitrageurs in asset pricing (e.g., Gromb and Vayanos (2002), Liu and Longstaff (2004), Basak and Croitoru (2006), Brunnermeier and Pedersen (2009), Kondor (2009), He and Krishnamurthy (2013), Lou and Polk (2013), Kondor and Vayanos (2013), Hanson and Sunderam (2014), Chinco (2014), Drechsler and Drechsler (2014), and Stambaugh, Yu, and Yuan (2012, 2014), and Kozak, Nagel, and Santosh (2018)). These studies focus on the impact of arbitrageurs in contagion, risk sharing, liquidity, portfolio choice, limit of arbitrage, the relation between the cost of shorting and anomaly returns, and so on, while our paper shed light on the effect of discovery by arbitrageurs. Cho (2018) highlights the role of arbitrage activity in determining the cross-section of betas.

Our paper is also related to the large literature on comovement. Existing studies demonstrate that comovement appear excessively *high* relative to fundamentals due to behavioral or friction-based reasons, when arbitrage is limited (e.g., Shiller (1989), Karolyi and Stulz (1996), Daniel, Hirshleifer, and Subrahmanyan (2001), Barberis and Shleifer (2003), Barberis, Shleifer, and Wurgler (2005), Peng and Xiong (2006), Green and Hwang (2009), Bartram, Griffin, Lim, Ng (2015), Da and Shive (2016)). We instead demonstrate that commovement can be excessively *low* among certain assets due to arbitrage trading. This complements prior studies such as Kyle and Xiong (2001), which shows that arbitrageurs can induce excessively high comovements among assets especially during periods of high volatility.

There is an extensive literature on anomalies, exploring explanations that are consumption-based (e.g., Bansal, Dittmar, and Lundblad (2005)), investment-based (e.g., Hou, Xue, Zhang (2014)), institution-based (e.g., Vayanos and Woolley (2013)), and behavioral-based (e.g., Baker and Wurgler (2006)). See Harvey, Liu, and Zhu (2016) for a comprehensive list. While these explanations generally abstract away from the discovery aspect, we take it seriously and formally analyze its consequences.

Finally, our paper adds to the recent growing interest in meta analysis on the systematic patterns of a large number of anomalies (e.g., Stambaugh, Yu, and Yuan (2012), Harvey, Liu, and Zhu (2016), McLean and Pontiff (2016), Green, Hand and Zhang (2017, GHZ), Hou, Xue, and Zhang (2017), Yan and Zheng (2017), Dong, Feng, and Sadka (2017)).

The rest of the paper is as follows. Sections 2 presents a model of risk-based anomaly, Section 3 analyzes a mispricing-based anomaly. Empirical analysis is reported in Section 4, and Section 5 concludes. The numerical algorithm and proofs are in the appendix.

2 A model of the discovery of a risk-based anomaly

Consider a two-period model, with time $t = 0, 1, 2$. Trading takes place at $t = 0, 1$, and consumption occurs at $t = 2$. There is one risk-free asset, and its interest rate is normalized to 0. There are two risky assets, asset 1 and asset 2, each of which is a claim to a single cash flow at $t = 2$. There is a continuum of identical investors, with a population size of one. At $t = 0$, investors are endowed with one unit of each asset and k dollars cash.

Asset i, for $i = 1, 2$, is a claim to a cash flow D_i at time $t = 2$. Moreover, D_1 and D_2 are independent from each other and have the same *ex ante* distribution at $t = 0$. Specifically, for $i = 1, 2$, we have

$$D_i = \mu_{i,1} \times \mu_{i,2}, \tag{1}$$

where $\mu_{i,1}$ and $\mu_{i,2}$ are random variables that will be realized at time $t = 1$ and $t = 2$, respectively. Moreover, $\mu_{i,t}$, for $i = 1, 2$ and $t = 1, 2$, are independent across i and t, and have the same binary distribution:

$$\mu_{i,t} = \begin{cases} \mu + \sigma & \text{with probability } p, \\ \mu - \sigma & \text{with probability } 1 - p, \end{cases} \tag{2}$$

where $\mu > \sigma > 0$, and $0 < p < 1$.

For $i = 1, 2$, and $t = 0, 1, 2$, we use $P_{i,t}$ to denote the price of asset i at time t, which will be determined endogenously in equilibrium. At $t = 2$, asset prices are pinned down by the final cash flow: $P_{i,2} = D_i$. We denote the gross return of asset i at time t, for $t = 1, 2$, as

$$r_{i,t} \equiv \frac{P_{i,t}}{P_{i,t-1}}.$$

2.1 Hedging demand

Investors are endowed with a nontradable asset (e.g., labor income), which is a claim to a cash flow ρD_1 at $t = 2$, with $\rho \geq 0$. That is, this endowment is perfectly correlated with the payoff from asset 1. Denote investors' wealth, excluding their nontradable endowment, at time t as W_t for $t = 0, 1, 2$. If investors allocate a fraction $\theta_{i,t}$ of W_t to asset i at time t, for $i = 1, 2$ and $t = 0, 1$, their wealth dynamic is given by

$$W_{t+1} = W_t \left[\sum_{i \in \{1,2\}} \theta_{i,t} r_{i,t+1} + \left(1 - \sum_{i \in \{1,2\}} \theta_{i,t} \right) \right], \tag{3}$$

with $W_0 = k + P_{1,0} + P_{2,0}$. Investors' objective is to choose $\theta_{i,t}$, for $i = 1, 2$, and $t = 0, 1$, to

$$\max_{\theta_{i,t}} E_0 \left[\log \left(W_2 + \rho D_1 \right) \right], \tag{4}$$

subject to (3). Investors find asset 1 riskier because its return is correlated with their endowment. As we will see later, due to this hedging demand, asset 1 has a lower price and a higher expected return in equilibrium. We will label this return pattern as an "anomaly," because when an econometrician observes the return data alone, he would not be able to explain it by CAPM. We, of course, do not take this formulation literally. Instead, the above formulation is meant to capture the essence of risk-based anomalies in a reduced form, i.e., asset 1 has low returns during "bad times."

2.2 Arbitrageurs

Traditional risk-based explanations of anomalies abstract away from the *discovery aspect*. Let us use the value premium as an example. By definition, the "discovery" of the value premium in Basu (1983) should make at least *some* market participants aware of the return pattern for the first time, unless one believes Basu was actually the last person to find out about the return pattern. In traditional risk-based models of the value premium, however, *all* investors knew about the value premium even before the discovery in Basu (1983). That is, this traditional approach does not take into account the effect of discovery, which is exactly the focus of our paper. That is, we analyze the fact that the discovery of the anomaly informs some agents about the return pattern for the first time. For convenience, we call those agents "arbitrageurs" to highlight that their risk exposure is different from that of the previously-described "investors."

Specifically, there is a continuum of identical arbitrageurs, with a population size of one. Their aggregate endowment at $t = 0$ is $W_0^a \geq 0$ dollars in cash. Importantly, they do not have the hedging demand that investors have, perhaps because arbitrageurs have a different labor income profile. To analyze the discovery effect across anomalies, we assume that arbitrageurs have access to another investment opportunity, which presumably exploits existing anomalies (say, e.g., currency carry trade). This opportunity is not available to the investors described earlier, perhaps because those investors do not have the expertise to analyze and implement the strategy. We call this existing anomaly "asset e," and assume its gross return at $t = 1, 2$ is

$$r_{e,t} = \begin{cases} \mu_e + \sigma_e, & \text{with probability } p_e, \\ \mu_e - \sigma_e, & \text{with probability } 1 - p_e, \end{cases}$$

where $\mu_e > \sigma_e > 0$, and $0 < p_e < 1$. Moreover, $r_{e,t}$ is assumed to be independent from $\mu_{i,t}$. That is, the fundamentals of assets 1 and 2 are independent from the existing anomaly—asset e.

For simplicity, we assume that the return of the existing anomaly $r_{e,t}$ is exogenously given. This simplification shuts down the effect of the discovery on the returns of existing anomalies.

This effect, however, is going to be small if the amount of the capital attracted by this new anomaly is small relative to the aggregate arbitrage capital attracted by all existing anomalies.[4]

2.3 Discovery effect

To analyze the discovery effect, we compare the equilibria across the following two economies. In the first (pre-discovery) economy, arbitrageurs are not aware of the anomaly (i.e., that assets 1 and 2 have the same fundamentals but different prices at $t = 0$). Hence, they invest in asset e, but not in assets 1 or 2. In the second (post-discovery) economy, arbitrageurs become aware of the anomaly and start exploiting it, as well as investing in the existing anomaly—asset e. To capture this, we assume that arbitrageurs take a long-short strategy in the two assets so that they can exploit the anomaly and stay "market neutral."[5] Specifically, we use $\theta^a_{i,t}$ to denote the fraction of arbitrageurs' wealth that is invested in asset $i = 1, 2$, at time $t = 0, 1$. A market-neutral strategy is such that, for $t = 0, 1$,

$$\theta^a_{1,t} + \theta^a_{2,t} = 0. \tag{5}$$

Let us use $\theta^a_{e,t}$ to denote the fraction of arbitrageurs' wealth that is invested in asset e at time $t = 0, 1$. Then, arbitrageurs' wealth dynamic is given by

$$W^a_{t+1} = W^a_t \left[\sum_{i \in \{1,2,e\}} \theta^a_{i,t} r_{i,t+1} + \left(1 - \sum_{i \in \{1,2,e\}} \theta^a_{i,t} \right) \right], \tag{6}$$

for $t = 0, 1$. Their objective is to choose $\theta^a_{i,t}$ for $i = 1, 2, e$, and $t = 0, 1$, to

$$\max_{\theta^a_{i,t}} E_0 \left[\log \left(W^a_2 \right) \right], \tag{7}$$

subject to (5) and (6).

In the pre-discovery economy, arbitrageurs are on the sidelines and have no impact on the markets for assets 1 and 2.[6] Hence, the equilibrium can be defined as follows. The pre-discovery

[4]Of course, our simplification may miss some subtle dynamics. For example, one might conjecture that investors may substitute between major anomalies, and generate negative correlation among them. Asness, Moskowitz, and Pedersen (2013) document a negative correlation between value and momentum returns. These specific dynamics are beyond the scope of this paper.

[5]This assumption is made so that arbitrageurs focus on exploiting the anomaly. Alternatively, we can simply assume that after the discovery, arbitrageurs become aware of the existence of assets 1 and 2. Under this alternative assumption, however, arbitrageurs will not only take a long-short position in the two assets, but also start investing in both assets. The latter will simply push up the prices of both assets. We are not interested in analyzing this latter effect. Moreover, in the value premium example, for instance, it seems more natural to think that, after the discovery of the value premium, hedge funds start buying value stocks and shorting growth stocks, rather than hedge funds becoming aware of the existence of both value and growth stocks and starting to buy both of them.

[6]This assumption perhaps resembles the preference of hedge funds, who attempt to deliver market neutral returns, and so have little interest in assets 1 and 2 before the discovery. Another reason is that hedge funds may choose to self-impose restrictions on their investment opportunity set (He and Xiong (2013)).

competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and investors' portfolios ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$), such that investors' portfolios optimize (4), and markets clear, i.e., for $i = 1, 2$ and $t = 0, 1$,

$$W_t \theta_{i,t} = P_{i,t}. \tag{8}$$

Similarly, the post-discovery competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and portfolios of investors and arbitrageurs ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$; and $\theta^a_{i,t}$ for $t = 0, 1$, $i = 1, 2, e$), such that investors' portfolios optimize (4), arbitrageurs' portfolios optimize (7), and markets clear, i.e., for $i = 1, 2$ and $t = 0, 1$,

$$W_t \theta_{i,t} + W^a_t \theta^a_{i,t} = P_{i,t}. \tag{9}$$

The implicit assumption is that arbitrageurs do not have any hedging demand in asset 1 or 2. Moreover, after the discovery, they know that the cause of the anomaly is investors' hedging demand. These are simplifying assumptions. What is necessary is that arbitrageurs have less hedging demand in asset 1 than investors. Finally, even if arbitrageurs do not know the true cause of the anomaly, they will still invest in it, and the main implications in this alternative model remain similar to those in our current setup.[7]

2.4 Equilibrium

Proposition 1 (Pre-discovery) *The pre-discovery equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$ can be characterized by equation (8) and*

$$E_t \left[\frac{r_{i,t+1} - 1}{W_{t+1} + \rho P_{1,t+1}} \right] = 0, \text{ for } i = 1, 2,\ t = 0, 1. \tag{10}$$

Moreover, in this equilibrium, we have $P_{1,0} < P_{2,0}$.

The above proposition illustrates the anomaly: Although both assets have the same fundamentals *ex ante*, they have different prices and hence different future expected returns. Due to their endowment, investors find asset 1 more risky than asset 2, leading to a lower price for asset 1. We label this as an anomaly because if an econometrician had only the price data, he would find the return pattern puzzling. This is similar to the anomalies we see in the literature. For example, the value premium is a puzzle if one looks at the return data alone. Risk-based models try to explore the idea that value stocks have a higher exposure to certain risk factors, which is

[7]See Brennan and Xia (2001) for an analysis of this intuition in the portfolio choice context.

similar to the reduced-formulation of the hedging demand in our model. While traditional risk-based models focus on the detailed analysis of the exact mechanism through which the hedging demand arises, they assume away the discovery aspect since all investors know the return pattern all along. In contrast, we are not interested in the details of the hedging demand, but focus on the analysis of the consequences of the discovery.

The following proposition characterizes the post-discovery equilibrium.

Proposition 2 (Post-discovery) *The post-discovery equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$ and $\theta^a_{i,1}$ can be characterized by equations (5), (9), (10), and for $t = 0, 1$,*

$$\begin{aligned} E_t\left[\frac{r_{1,t+1} - r_{2,t+1}}{W^a_{t+1}}\right] &= 0, \\ E_t\left[\frac{r_{e,t+1} - 1}{W^a_{t+1}}\right] &= 0. \end{aligned}$$

Since arbitrageurs are not exposed to the endowment risk that investors have, they find the anomaly an attractive investment opportunity, and buy asset 1 and short asset 2. For convenience, we call the return from this long-short portfolio, $r_{1,1} - r_{2,1}$, the "anomaly return."

To analyze the discovery effect, we will compare the post-discovery equilibrium in Proposition 2 with the pre-discovery equilibrium in Proposition 1.[8] In particular, following the algorithm in Appendix A, we solve both equilibria numerically. The baseline parameter values are summarized in Table 1. In the following numerical analysis, we vary only one parameter at a time to examine the effects of the discovery. We have also repeated our numerical analyses for other parameter values, and none of the following qualitative results are specific to the chosen parameters.

2.5 Anomaly magnitude

Figure 1 illustrates the effects of discovery on the expected anomaly returns. The dashed line represents the size of the anomaly (i.e., the expected anomaly return $E_0[r_{1,1} - r_{2,1}]$) before the discovery. Since arbitrageurs have no influence on the markets for assets 1 and 2 before the discovery, the dashed line is flat: The expected anomaly return is around 5.5% regardless of

[8] The equation system in Proposition 2 is highly nonlinear and we have not been able to establish the existence and uniqueness of their solutions. However, we have always been able to solve the equation system numerically, and the solution appears to be unique. One might be somewhat surprised that the simple two-period structure in our model does not allow for a closed-form solution. In fact, the wealth effect in our model has similar complexity as that in the continuous-time model in Xiong (2001), which also heavily relies on numerical analysis. As noted in Gromb and Vayanos (2002), a two-period model of arbitrageurs and investors with a wealth effect is not as tractable as its appearance suggests (page 381). In a recent study, Kondor and Vayanos (2013) gain more tractability by simplifying investors' decisions.

arbitrageurs' wealth.

After the discovery, arbitrageurs start exploiting the opportunity, reducing the expected anomaly return. As shown by the solid line in Panel A, the post-discovery expected anomaly return is lower than that in the pre-discovery case (i.e., the solid line is below the dashed line). In the case $W_0^a = 2$, for example, the discovery reduces the expected anomaly return from 5.5% to 5%.

The plot also shows that the effect of discovery is stronger when arbitrageurs have more wealth. For example, in the case $W_0^a = 5$, the discovery reduces the expected anomaly return from 5.5% to 4%. The discovery effect disappears when $W_0^a = 0$. One can think of this $W_0^a = 0$ case as representing the traditional modeling approach, where discovery does not change the set of investors who are aware of the anomaly.

Panels B and C demonstrate the effects of arbitrageurs' existing investment opportunity (i.e., asset e). If arbitrageurs' existing strategy is more attractive (i.e., μ_e is higher, or σ_e is lower), they will allocate less capital to exploit the new anomaly and so its expected return will drop less. As shown in Panels B and C, after the discovery of an anomaly, its expected return is increasing in μ_e and decreasing in σ_e.

2.6 Correlation among anomaly returns

By the construction of our model, before the discovery, the anomaly return $r_{1,1}-r_{2,1}$ is independent of the return of the existing anomaly $r_{e,1}$. How does the discovery affect the correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$?

Intuitively, after the discovery of an anomaly, arbitrageurs start exploiting it, as well as the existing anomaly, asset e. This creates a correlation through the wealth effect. Suppose the return from asset e is unexpectedly high one period. This increases the wealth of these arbitrageurs. Everything else being equal, they will allocate more investment to the newly discovered anomaly. This higher investment pushes up the price of asset 1 and pushes down the price of asset 2, leading to a high anomaly return $r_{1,1} - r_{2,1}$. Similarly, an unexpectedly low return from asset e leads to a low anomaly return. That is, the wealth effect increases the correlation between the newly discovered anomaly return and the return from the existing anomaly.

The above intuition is illustrated in Figure 2. Panel A plots the correlation coefficient between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. Before the discovery, as illustrated by the dashed line, the correlation is 0. In contrast, the post-discovery correlation, shown by the solid line, is positive. The only exception is the case $W_0^a = 0$, where the arbitrageurs have no wealth.

This discovery effect (i.e., the change in the correlation across the pre- and post-discovery cases) is initially increasing in the size of arbitrage capital W_0^a, and is not monotonic. This is because arbitrageurs have two effects on the correlation. The first is the aforementioned wealth effect, which increases the correlation. The second is that as arbitrage capital increases, the prices of assets 1 and 2 are more driven by their fundamentals. This reduces the correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. When the size of arbitrage capital is sufficiently large, the second effect dominates, and hence a further increase in arbitrage capital reduces the correlation.

The above intuition is further illustrated in Panels B and C. In particular, when arbitrageurs have a larger position in asset e (due to a higher μ_e or a lower σ_e), their wealth becomes more sensitive to its realized return $r_{e,1}$. This leads to a stronger wealth effect, i.e., the discovery has a stronger effect in generating the correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. In Panel B, for example, as the expected return from asset e increases (i.e., a higher μ_e), it leads to a higher correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. Similarly, in Panel C, as the volatility of asset e increases (i.e., a higher σ_e), it leads to a weaker wealth effect and a lower correlation.

2.7 Correlation between assets 1 and 2

Our model shows that the discovery of an anomaly reduces the correlation coefficient between the returns of assets 1 and 2. The intuition is as follows. After the discovery, arbitrageurs long asset 1 and short asset 2 to exploit the anomaly. Now, suppose arbitrageurs' wealth increases due to, say, a high return from their investment in asset e. They will buy more of asset 1 and sell more of asset 2. This increases asset 1's return but decreases asset 2's return. Similarly, when arbitrageurs' wealth decreases, they will unwind some of their positions in the long-short portfolio. That is, they will sell asset 1 and buy asset 2, decreasing asset 1's return but increasing asset 2's return. In both cases, arbitrageurs' wealth shocks push the returns of the two assets to opposite directions, which reduces the correlation between the returns of assets 1 and 2.

This intuition is illustrated in Figure 3. The dashed line in Panel A is for the pre-discovery correlation between assets 1 and 2. Since arbitrageurs are on the sidelines before the discovery, their wealth level W_0^a does not affect the correlation. Hence, the dashed line is flat. The post-discovery case is represented by the solid line. It is below the dashed line, suggesting that the discovery reduces the correlation between assets 1 and 2. It also shows that the larger the size of arbitrage capital, the larger the reduction in the correlation.

The above intuition further suggests that the discovery effect is stronger when arbitrageurs' wealth is more volatile. To illustrate this intuition, we plot the correlation between assets 1 and 2 against arbitrageurs' wealth volatility, which is an endogenous variable. Specifically, we vary arbitrageurs' wealth volatility by changing μ_e from 1.1 to 1.46.[9] The solid line in Panel B shows that after the discovery, the correlation between assets 1 and 2 is decreasing in arbitrageurs' wealth volatility. In contrast, this relation does not hold before the discovery, as shown by the dashed line.

2.8 Diversification benefits

An economic consequence of the reduction in the correlation between assets 1 and 2 is that arbitrageurs' trading leads to diversification benefits for passive investors, who hold the market portfolio of assets 1 and 2. To see this, we plot the volatility of an equal-weighted portfolio consisting of assets 1 and 2 in Figure 4. As expected, we find that the portfolio volatility decreases following the anomaly discovery.

In particular, Panel A of Figure 4 shows that the diversification benefits, i.e., reductions in market volatility, are stronger when arbitrageurs' wealth is higher, since the reduction in the post-discovery correlation between assets 1 and 2 increases with the size of arbitrage capital. By the same token, the diversification benefits are also stronger when arbitrageurs' wealth volatility is higher, as shown in Panel B of Figure 4.

3 Mispricing-based anomaly

We now analyze a model in which the anomaly is caused by investors' behavioral bias. Specifically, we modify the previous model by setting $\rho = 0$; that is, there is no hedging demand. The fundamentals of the two assets are still given by (1) and (2). However, investors are biased about asset 1 and believe that for $t = 1, 2$,

$$\mu_{1,t} = \begin{cases} \mu + \sigma & \text{with probability } p - b, \\ \mu - \sigma & \text{with probability } p + b, \end{cases} \tag{11}$$

where $0 \leq b < p$. That is, investors underestimate asset 1's expected cash flow, and b measures the degree of the bias. In contrast, their belief about asset 2 is correct.

[9] Qualitatively similar results can be generated by varying σ_e instead.

Investors' objective is to choose $\theta_{i,t}$, for $i = 1, 2$, and $t = 0, 1$, to

$$\max_{\theta_{i,t}} E_0^* \left[\log \left(W_2 \right) \right], \tag{12}$$

subject to (3), where $E_0^* [\cdot]$ indicates that the expectation is taken under the biased belief in (11). Arbitrageurs have correct beliefs, and their objective is given by (7), as in the previous section.

This formulation is meant to capture the essence of mispricing-based interpretations of anomalies in a reduced form. For instance, in the value premium example, Lakonishok, Shleifer, and Vishny (1994) argue that investors are overly enthusiastic about glamorous growth stocks and have a low demand for value stocks. Similarly, in our model, investors underestimate the payoff from asset 1 and so have a low demand.

Similar to the case of the risk-based anomaly, in the pre-discovery case, arbitrageurs have no influence on the markets for assets 1 and 2. The competitive equilibrium for this case is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and investors' portfolios ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$), such that investors' portfolios optimize (12), and markets clear as in (8).

The post-discovery competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and portfolios of investors and arbitrageurs ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$; and $\theta_{i,t}^a$ for $t = 0, 1$, $i = 1, 2, e$), such that investors' portfolios optimize (12), arbitrageurs' portfolios optimize (7), and markets clear as in (9).

What is implicitly assumed here is that the discovery does *not* affect investors' bias b. That is, the bias is systematic and deeply rooted, and investors do not adjust their behavior after the discovery of the anomaly. This assumption is made for simplicity. Alternatively, if the bias is partially reduced after the discovery, the results remain qualitatively similar.

Proposition 3 *The pre-discovery equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$ can be characterized by (8), and for $i = 1, 2$, $t = 0, 1$,*

$$E_t^* \left[\frac{r_{i,t+1} - 1}{W_{t+1}} \right] = 0. \tag{13}$$

The post-discovery equilibrium prices $P_{i,t}$ and portfolio choices ($\theta_{i,t}$, and $\theta_{i,1}^a$) can be characterized by equations (5), (9), (13), and for $t = 0, 1$,

$$E_t \left[\frac{r_{1,t+1} - r_{2,t+1}}{W_{t+1}^a} \right] = 0, \tag{14}$$

$$E_t \left[\frac{r_{e,t+1} - 1}{W_{t+1}^a} \right] = 0. \tag{15}$$

Similar to the risk-based case in the previous section, investors have a lower demand for asset 1 than for asset 2. The only difference is the motivation. In the risk-based case, the motivation is to hedge, while in the mispricing-based case, the motivation is investors' wrong belief. To compare the post-discovery return dynamic across the risk-based case and the mispricing-based case, we set $b = 0.055$ and adopt all other parameters from Table 1. We choose this value for b so that, before the discovery, the expected anomaly returns are the same across the two cases.

Panel A of Figure 5 shows that the post-discovery performance of a mispricing-based anomaly is similar to that of a risk-based anomaly. The solid and dashed lines represent the post-discovery expected anomaly return for the risk- and mispricing-based cases, respectively. The pre-discovery expected anomaly return for both cases is flat at around 5.5% (we omitted this flat line). The plot shows that the discovery of an anomaly reduces its expected return regardless of whether the anomaly is caused by risk or mispricing. Moreover, both lines are downward sloping, implying that the more arbitrage capital (W_0^a), the stronger the effect. Panel B shows that, for both the risk- and mispricing-based cases, the discovery of an anomaly increases the correlation between its return and the existing anomaly return. Even the non-monotonic pattern is similar across the two cases. Finally, Panel C shows that the discovery of the anomaly reduces the correlation between assets 1 and 2 for both risk- and mispricing-based cases. Moreover, this correlation is decreasing in arbitrageurs' wealth level W_0^a in both cases.

As the discovery effect can be similar across risk- and mispricing-based anomalies, in the next section we will include in our empirical analysis all the anomalies for which data is available, without worrying about the sources of the anomalies.

4 Empirical Analysis

Our risk-based model in Section 2 and mispricing-based model in Section 3 share the same two sets of predictions on *asset prices*. First, the discovery of an anomaly reduces its magnitude and increases its correlation with existing anomalies. Second, the discovery of an anomaly reduces the correlation between assets 1 and 2. The first prediction is consistent with existing empirical evidence (e.g., McLean and Pontiff (2016)). The second prediction is new to the literature, and we empirically test them in this section. Moreover, the two versions of our models also share their predictions on *arbitrageurs' trading*, which we will also empirically examine in this section.

4.1 Data

We build our anomaly variables by starting from all the anomaly characteristics listed in Green, Hand and Zhang (2017, GHZ), which are required to be calculable entirely from the widely available CRSP, Compustat and/or I/B/E/S data. We focus on the "main-effect signals," i.e., characteristics that are interactions between other characteristics are excluded, to avoid confounding effects. We further require that the anomaly sorting variables are continuous so that decile portfolios can be formed for calculating our excess correlation measure. As such, dummy variable-based anomalies are excluded. We then add the anomalies that are not included in GHZ but are studied in Stambaugh, Yu, and Yuan (2012). This results in a final list of 99 anomalies.

For each month t's return we calculate characteristics as they were at the end of month $t-1$, assuming that annual accounting data are available at the end of month $t-1$ if the firm's fiscal year ended at least six months before the end of month $t-1$, and that quarterly accounting data are available at the end of month $t-1$ if the fiscal quarter ended at least four months before the end of month $t-1$. We take monthly stock returns from CRSP and include delisting returns following Shumway and Warther (1999). Our sample for the monthly anomaly decile portfolio returns and risk-free asset returns is from 1963 to 2015.

We use the publication date as a proxy for the discovery time. For unpublished anomalies, we use the date of the latest working paper. It is not obvious how to choose the "discovery time" for each anomaly. The decision is necessarily subjective to some extent. Suppose we choose the publication time of the first study on the anomaly. It is possible that practitioners have known and exploited the anomaly before that. The essence of the "discovery time" in our model is the time when a large number of arbitrageurs start exploiting the anomaly. One might suspect that it may take some time after arbitrageurs become aware of an anomaly for them to be convinced and start exploiting it. Moreover, the first publication might not be the one that generates most attention. Therefore, for an anomaly that has multiple papers focusing on it as the main effect in the paper, we choose the publication date of the most cited paper. Hence, one should not take the literal interpretation that those anomalies were secrets before the publication time and became public information afterwards. Rather, it is natural to expect that the publicity attracts more attention from arbitrageurs after the publication date as compared to before the publication date. Arbitrageurs are therefore more likely to trade on it. We will empirically examine this prediction. We also note that in the early 1990s, when the hedge fund industry grew rapidly and hence more arbitrage capital are likely to start exploiting those anomalies. This is also consistent with our notion of discovery, which operates through the pricing effects of arbitrage capital. That is, the discovery time for an anomaly is the time after which significantly more arbitrage capital starts exploiting the anomaly.

Table 2 provides the summary statistics of the anomalies. Among the 99 anomalies, 87% of them (86) have a return with the absolute value of the t-statistic above 1.5. The average long-short return of these anomalies are 70 bps per month. These results confirm that the anomaly findings documented in their original studies. The average correlations between them is low (0.05), consistent with other studies reporting a similar number (e.g., McLean and Pontiff (2016), Green, Hand and Zhang (2017, GHZ)).[10] Internet Appendix Table A.1 provides further details of the 99 individual anomalies.

We obtain the monthly hedge fund returns and assets under management (AUM) from TASS for the period of 1981–2015.[11] Since we examine anomalies in the U.S. equity market, we only keep U.S. equity funds by using the filters used in prior studies (e.g., Chen, Han, and Pan 2017). Then, we compute the percentage change in AUM for each fund and aggregate them into the value-weighted average of percentage AUM change of all funds. For each month during 1986–2015, hedge fund wealth volatility is calculated as the standard deviation of this aggregate AUM percentage changes during the previous 5 years, excluding the current month t. The summary statistics for hedge fund wealth volatility are reported in Table 2**.** This series has a mean of 0.08 and a standard deviation of 0.10.

To measure arbitrageurs' trading activities, we construct two proxies. First, we utilize the classification in Agarwal, Jiang, Tang, and Yang (2013) which combines the information in the 13F institutional holdings data and hedge fund name information from a union of 5 major hedge fund databases to identify the hedge funds in 13F. The 13F holdings data cover by far the largest number of institutional investors: all institutional investment managers (including foreign investors) that have investment discretion over \$100 million or more in Section 13(f) securities (mostly publicly traded equity) are required to disclose their quarter-end holdings in these securities. A 13F-filing institution is classified as a hedge fund if its major business is sponsoring/managing hedge funds according to the information revealed from a range of sources, including the institution's own websites, SEC filings, industry directories and publications, and news article searches. A Form 13F is filed at the "management company" rather than at the "portfolio" or at the indi-

[10] 90% of the anomalies are based on published papers. Removing anomalies based on working papers do not change the results of the paper.

[11] 1981 is the first year TASS has comprehensive TNA data.

vidual fund level. We identify the hedge fund holdings for the period in which we have the hedge fund AUM data. Our final sample consists of 942 unique hedge funds.

Second, we utilize the monthly short interest data for NYSE, AMEX, NASDAQ stocks from COMPUSTAT to proxy for arbitrageurs' short positions. The short interest data for NASDAQ stocks are available after 2000. We therefore focus on short selling after 2000. A stock's short interest in a month is the total number of uncovered shares sold short for transactions settled on or before the 15th of the month, normalized by the total number of shares outstanding, which is obtained from CRSP.

4.2 Correlation

Our model implies that the discovery of an anomaly reduces the correlation between the long and short legs of the anomaly. To test this prediction, we examine whether the correlation coefficient between the excess returns of deciles 1 and 10 decreases after the discovery of the anomaly.[12] We estimate the monthly correlation coefficient between the excess returns from deciles 1 and 10 based on a 5-year rolling window of returns, including the current month. To formally test whether the correlations decrease after discoveries, we adjust the correlations between deciles 1 and 10 by the correlations between deciles 5 and 6. For each anomaly i, we compute the *excess correlation* as

$$X_{i,t} \equiv \rho_{i,t}^{1,10} - \rho_{i,t}^{5,6}, \tag{16}$$

where $\rho_{i,t}^{1,10}$ is the correlation coefficient between the monthly excess returns of deciles 1 and 10 of anomaly i during the five years prior to month t, and $\rho_{i,t}^{5,6}$ is similarly defined for deciles 5 and 6.

This adjustment controls for a potential time trend for the correlation among stocks. The motivation is the following. To exploit the anomaly, arbitrageurs are likely to take larger long-short positions in deciles 1 and 10 than in deciles 5 and 6. Hence, the correlation between deciles 5 and 6 may share a common time trend with the correlation between deciles 1 and 10, but should be subject to a weaker discovery effect. As shown in Table 2, the excess correlation has a mean of -0.10, and standard deviation of 0.14.

We regress the excess correlation $X_{i,t}$ on $Discovery_{i,t}$, a dummy variable that takes the value of 0 before the discovery of anomaly i and 1 afterwards. In all regressions, we control for anomaly

[12] Lou and Polk (2013) uses the high-frequency correlation among stocks to infer the size of arbitrage capital. The economic mechanism is quite different. In their setup, higher correlations among *stocks within Decile 10 (or Decile 1)* imply a *larger* arbitrage capital size. However, in our setup, higher low-frequency correlations *between decile 1 and 10 portfolios* imply a *smaller* arbitrage capital size.

fixed effects. The standard errors are clustered by anomalies.[13]

The first column of Table 3 shows that the coefficient for the discovery dummy is -0.05, with a t-statistic of -6.39. This suggests that the excess correlation decreases by 5% on average after the discovery of an anomaly. This is a sizable reduction as it represents 36% of the standard deviation of the correlation coefficient.

In the above regression, all anomalies are treated equally. However, one would expect that some anomalies are more likely to attract the attention from arbitrageurs than others. As a proxy for arbitrageurs' attention to each anomaly, we obtain Google citation count, as of October 2016, of the studies that first discovered the anomaly. Admittedly, this measure is likely to be quite imprecise, and hence is likely to be biased against any statistical significance in our tests. Following Ang, Hodrick, Xing, and Zhang (2006), we use weighted least square to estimate the discovery effects, where the weights are based on the citation counts of the studies that discovered the anomalies. As shown in column 3, the coefficient for the discovery dummy is -0.09 ($t = -9.11$), representing 64% of the standard deviation of the correlation coefficient. That is, consistent with our intuition, the discovery effect is much stronger when the anomaly publication is more widely cited.

4.3 The role of arbitrageurs

In our model, the discovery effect operates through arbitrage trading: arbitrageurs' trading activity reduces the correlation between deciles 1 and 10. Hence, a direct test of this view is to examine whether this correlation is indeed related to arbitrageurs' activity.

Our model implies that the post-discovery correlation between deciles 1 and 10 of an anomaly is decreasing in the volatility of arbitrageurs' wealth. This prediction is the opposite of the implication from the conventional intuition that arbitrageurs' wealth tends to be more volatile when the market is more volatile (e.g., in a financial crisis). Since stocks tend to be more correlated when the market is more volatile, this conventional intuition implies that the correlation between deciles 1 and 10 should be positively related to the volatility of arbitrageurs' wealth.

To test our hypothesis, we need a proxy for the volatility of arbitrageurs' wealth. It is certainly impossible to directly observe the aggregate wealth of all arbitrageurs. As a compromise, we measure the wealth of one group of investors, who are often considered to be arbitrageurs in financial markets: hedge funds. The implicit assumption is that the volatility of the aggregate

[13] The results are also similar if we double-cluster standard errors by anomalies and time.

wealth of all hedge funds is positively correlated with the volatility of the total wealth of all arbitrageurs.

To test our hypothesis, we run a panel regression of the excess correlation X_t on the interaction between the discovery dummy and hedge fund wealth volatility. To ease the interpretation of the economic importance of the regression, we scaled the hedge fund wealth volatility by its standard deviation. The normalized variable is denoted $wealth\_vol_t$. Our focus is the interaction term $Discovery \times wealth\_vol_t$. Our model implies that the hedge fund wealth volatility reduces the correlation between deciles 1 and 10 *after* the discovery. That is, the coefficient for the interaction term should be negative. This is indeed the case. Column 3 of Table 3 shows that the coefficient for the interaction term is -0.05, with a t-statistic of -2.37. This means that a one standard deviation increase in wealth volatility will result in 5% more reduction in correlation post discovery than before discovery. This magnitude is again sizable and represents 36% of the standard deviation of the correlation coefficient.

We also estimate the regression by citation-weighted least square. As shown in column 4, consistent with the interpretation that the discovery effect is stronger for influential discoveries, the coefficient for the interaction term $Discovery \times wealth\_vol_t$ become more than twice larger, and is -0.13 ($t = -3.00$). That is, a one standard deviation increase in wealth volatility will increase the discovery effect by 13%, which is 93% of the standard deviation of the correlation coefficient.

In summary, our evidence so far is consistent with the model prediction that the discovery of an anomaly reduces the correlation between the long and the short portfolios of the anomaly. Moreover, this effect is stronger when the aggregate asset under management by hedge funds is more volatile.

4.4 Diversification benefits

One economic consequence of this reduction in correlation is that it leads to diversification benefits to passive investors who hold both deciles 1 and 10. Note that the set of stocks in deciles 1 and 10 of an anomaly change over time. Hence, the diversification benefits are effectively shared by passive investors who hold the market portfolio.

In this section, we empirically examine this implication. Specifically, we estimate the volatility of the portfolio of deciles 1 and 10 based on a 5-year rolling window of monthly returns, including the current month. To control for the time trend in volatility in the market, we calculate the *excess* volatility as the volatility of deciles 1 and 10 over the volatility of the market portfolio, which is estimated similarly based on a 5-year rolling window.

We then regress this excess volatility on the *Discovery* dummy variable, with anomaly fixed effects. The first column of Table 4 shows that the coefficient for the discovery dummy is -0.007 with a t-statistic of -2.98 in the regression where anomalies are equal-weighted. The coefficient increases by almost three times to -0.02 with a t-statistic of -2.94 in the regression where anomalies are citation-weighted. The reductions in volatility are economically important as they represent 21% and 61% drop from the pre-discovery excess volatility (0.033) for the equal- and citation-weighted results, respectively

To examine the role arbitrageurs in the effect on diversification benefits, we run a panel regression of the excess volatility on the interaction term $Discovery \times wealth\_vol_t$. As shown in the column 3 of Table 4, the coefficient for the interaction term is -0.004 ($t = -2.83$). That is, a one standard deviation increase in wealth volatility would result in 0.4% more reduction in volatility, which represents a 12% drop from pre-discovery excess volatility. Finally, we also rerun the regression using citation-weighted least square. As shown in column 4, the effect becomes significantly stronger, both in economic magnitude and statistically. A one standard deviation increase in wealth volatility would result in 0.6% more reduction in volatility, representing a 18% drop from the pre-discovery excess volatility.

4.5 Arbitrage Trading

In this section, we directly examine the model implications on arbitrage trading. As in the previous section, we infer arbitrage trading from hedge fund holdings. Specifically, we compute the aggregate hedge fund purchase of each stock as the quarterly percentage change in their aggregate holdings in the stock. Then, for each anomaly, we use the average hedge fund purchase of decile 1 stocks minus that of decile 10 stocks as a proxy for arbitrageurs' trading on the anomaly. A positive value of this measure is consistent with arbitrageurs trading in the "right" direction to exploit the anomaly, while a negative value is consistent with them trading in the "wrong" direction. Table 2 shows that the average of our trading measure in the entire sample is insignificantly different from zero.

4.5.1 Discovery and Arbitrage Trading

To exploit an anomaly, arbitrageurs establish a long position in decile 1 and short position in decile 10. As the stocks in deciles 1 and 10 change over time, arbitrageurs need to re-balance their positions. That is, after the discovery of an anomaly, our arbitrageurs' trading measure should be sustained at a higher level. To test this prediction, we run a panel regression of our anomaly trading measure on the discovery dummy, with two lags of this trading measure as the control for its serial correlation. Consistent with the prediction, as shown in column 1 of Table 5, the coefficient of $Discovery$ is 0.02 ($t = 2.06$). That is, on average, the discovery of an anomaly is accompanied with an increase in hedge funds' anomaly trading equivalent to 2% of the total shares outstanding of the traded stocks. As shown in column 2, when we estimate the regression by citation-weighted least square, the effect is more than 3 times as large: the coefficient of $Discovery$ is 0.07 ($t = 7.58$). Therefore, the results suggest that discovery is accompanied with economically and statistically significant increase in hedge fund trading that exploits the anomaly.

The literature so far has been inconclusive on whether arbitrageurs exploit anomalies in the right direction. Ali, Chen, Yao, and Yu (2008) find that mutual funds in aggregate do not trade on the accrual anomaly. Edelen, Ince, and Kadlec (2016) shows that institutional investors trade 7 anomalies in the "wrong" direction on average. Calluzzo, Moneta, and Topaloglu (2017) shows the opposite results to Edelen, Ince, and Kadlec (2016) using 14 anomalies, particularly after anomalies' publication dates. In contrast, our evidence is based on a much more comprehensive set of anomalies. More importantly, we also further examine, in the next section, arbitrageurs' trading after wealth shocks, a main mechanism in our model.

4.5.2 Wealth Change and Arbitrage Trading

Our model suggests that after the discovery of an anomaly, arbitrage trading on the anomaly intensifies (reverses), when arbitrageurs' wealth increases (decreases). To test this, we run a panel regression of our anomaly trading measure on the interaction term of the discovery dummy and the lagged change in hedge fund wealth (scaled by the standard deviation of wealth change). Consistent with our prediction, as shown in column 3 of Table 5, the coefficient of the interaction term is 0.02 ($t = 2.81$). That is, after the discovery of an anomaly, a one standard deviation increase (decrease) in the aggregate asset under management by hedge funds leads to an increase (decrease) in their positions that exploit the anomaly by around 2% of the total shares outstanding of the stocks traded. We also run the citation-weighted least square regression, and, as shown in the last column, although the point estimate of the interaction coefficient remains similar, its t-statistic increases to 5.97. This evidence supports our model implication that after the discovery of an anomaly, arbitrageurs actively adjust their positions over time to exploit the anomaly.

4.5.3 Arbitrage Trading Inferred from Short Selling

To complement the above analysis based on hedge fund holdings in long positions, we construct an alternative arbitrage trading measure from short selling activities. Short interest includes all short positions and is not necessarily initiated by hedge funds. But given that mutual funds and pension funds are generally not allowed to short, the bulk of short positions are likely from hedge funds.

Specifically, for each anomaly, we compute the average of the change in short interest of decile-1 stocks and that of decile-10 stocks. The anomaly trading measure is the change in short interest of decile 10 minus that of decile 1. Hence, a positive value of this measure is consistent with arbitrageurs trading in the "right" direction to exploit the anomaly, while a negative value is consistent with them trading in the "wrong" direction.

We rerun the same panel regressions as in Table 5 using the short selling-based anomaly trading measure at the monthly frequency. As shown in column 1 of Table 6, the coefficient on the discovery dummy term is 0.04 (t = 2.15). That is, after the discovery of an anomaly, arbitrageurs increase their short selling in the direction of exploiting the anomaly by 4% of the shares outstanding. On citation-weighted basis, the estimate of the coefficient is 0.04 (t = 3.94). Similarly, we regress the short selling-based anomaly trading measure on the interaction term of the discovery dummy and the lagged change in hedge fund wealth scaled by the standard deviation of the wealth change. As shown in column 3, the coefficient of the interaction term is 0.03 (t = 2.28). That is, a one standard deviation increase (decrease) in the aggregate asset under management by hedge funds leads to 3% more (less) short interest in the direction of exploiting the anomaly in the post-discovery period. On the citation-weighted basis, the effect is even stronger. The estimate of the interaction coefficient is 0.07 (t = 3.36).

In summary, the evidence in this section further supports the arbitrage trading implications in our model. That is, after the discovery of an anomaly, arbitrageurs trade more to the direction of exploiting the anomaly. They also adjust their positions over time, establishing more positions to exploit the anomaly when their wealth increases, but unwind their positions when their wealth decreases.

5 Conclusion

We have analyzed a stylized model of anomaly discovery, which has implications for both *asset prices* and *arbitrageurs' trading*. Our model shows that consistent with existing evidence, the discovery of an anomaly reduces its magnitude and makes its returns more correlated with the returns from existing anomalies. Moreover, our model shows that the discovery of an anomaly reduces the correlation between the two extreme portfolios formed from the corresponding portfolio sorting for that anomaly, and that this effect is stronger when arbitrageurs' wealth is more volatile. One economic consequence of this reduction in correlation is that it leads to diversification benefits for passive investors who hold the market portfolio. We empirically test these new predictions for 99 anomalies, and find clear evidence consistent with our model predictions. Moreover, we also directly examine arbitrageurs' trading behavior. Consistent with our model predictions, we find that after the discovery of an anomaly, arbitrageurs increase (reverse) their positions in exploiting the anomaly when their wealth increases (decreases), further supporting the view that the discovery effects work through arbitrage trading.

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Appendix A. Numerical procedure

We follow the procedure described below to solve the model:

1. Take initial guesses for the total wealth for investors and arbitrageurs at $t = 1$: W_1 and W_1^a for the eight states at date 1.

2. For each of the eight states, take W_1 and W_1^a as given, solve for the portfolios ($\theta_{i,1}$ for $i = 1, 2$, and $\theta_{i,1}^a$ for $i = 1, 2, e$) and prices $P_{1,1}$ and $P_{2,1}$.

3. Take the prices $P_{1,1}$ and $P_{2,1}$ for the eight states in step 2 as given, solve for the $t = 0$ portfolios ($\theta_{i,0}$ for $i = 1, 2$, and $\theta_{i,0}^a$ for $i = 1, 2, e$) and prices $P_{1,0}$ and $P_{2,0}$.

4. Based on the portfolios in step 3 ($\theta_{i,0}$ for $i = 1, 2$, and $\theta_{i,0}^a$ for $i = 1, 2, e$) and the prices in steps 2 and 3 ($P_{1,0}$, $P_{2,0}$, and $P_{1,1}$, $P_{2,1}$ for all eight states at $t = 1$), calculate the investors' and arbitrageurs' updated wealth, W_1 and W_1^a, in the eight cases at $t = 1$.

5. Repeat steps 2 to 4 until the wealth, portfolios, and prices converge, i.e., for each variable, the difference between two iterations is no greater than 0.00005.

Appendix B. Proofs

Proof of Propositions 1 and 2

Due to the logarithmic preference, the maximization problem (4) is equivalent to maximizing the log wealth growth for each period. Hence, investors' first-order conditions are given by

$$E_t\left[\frac{r_{i,t+1} - 1}{W_{t+1} + \rho P_{1,t+1}}\right] = 0,$$

for $i = 1, 2, t = 0, 1$. Similarly, the arbitrageurs' optimization problem (7) can also be decomposed into a period-by-period optimization problem, and the first-order conditions are given by

$$\begin{aligned} E_t\left[\frac{r_{1,t+1} - r_{2,t+1}}{W_{t+1}^a}\right] &= 0, \\ E_t\left[\frac{r_{a,t+1} - 1}{W_{t+1}^a}\right] &= 0. \end{aligned}$$

Combining the above first-order conditions with the market-clearing conditions, we can characterize the equilibria in Propositions 1 and 2.

We now prove $P_{1,0} < P_{2,0}$ by contradiction. Suppose $P_{1,0} \geq P_{2,0}$. Note that investors' optimal portfolio in equilibrium is to hold one unit of both assets. Suppose an investor sells ϵ unit of asset 1 and buys ϵ unit of asset 2. Define his expected utility as

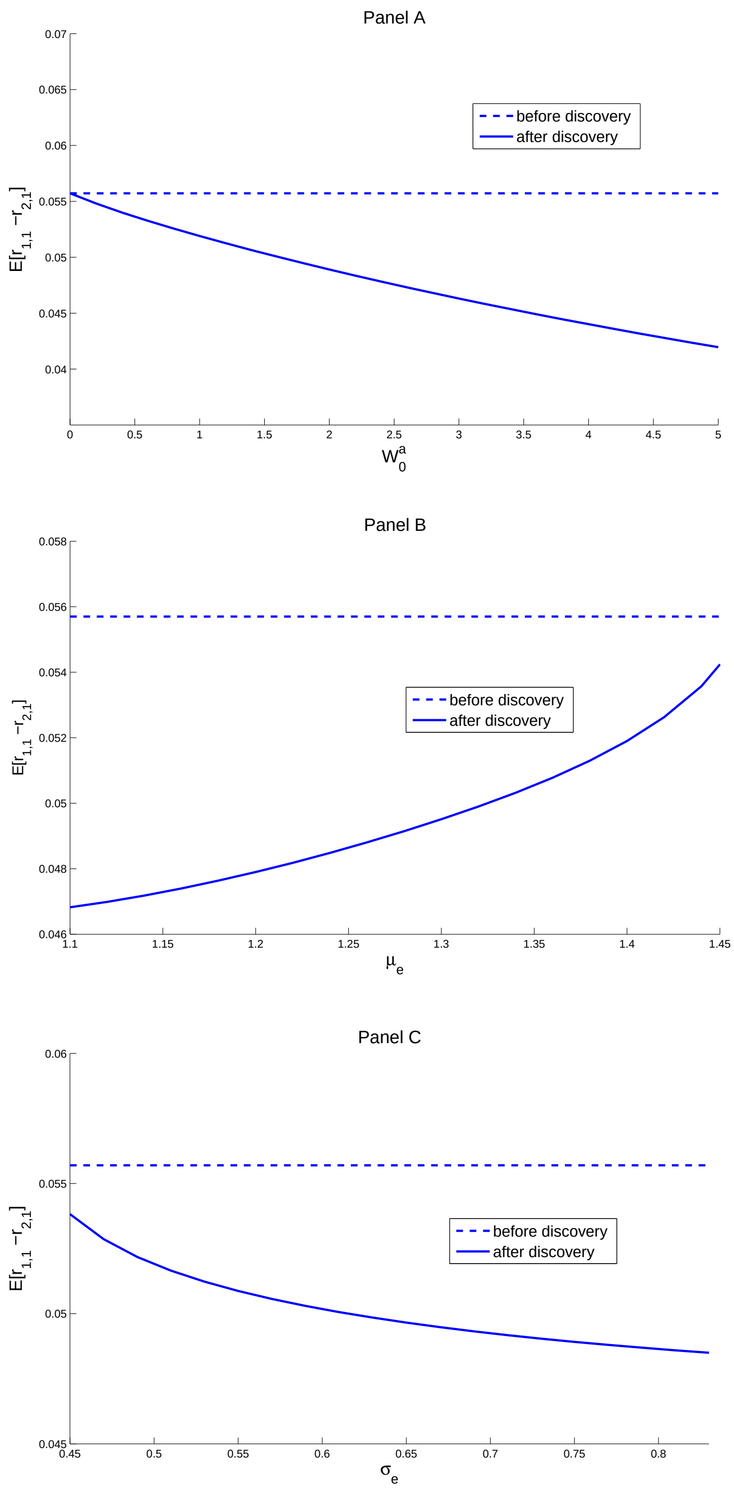
$$U(\epsilon) \equiv E_0[\log(k + (1 + \rho - \epsilon)D_1 + (1 + \epsilon)D_2)].$$

It is easy to see that $\frac{dU}{d\epsilon}|_{\epsilon=0} > 0$. That is, he can strictly improve his portfolio by selling ϵ unit of asset 1 and buying ϵ unit of asset 2. This leads to a contradiction.

Proof of Propositions 3

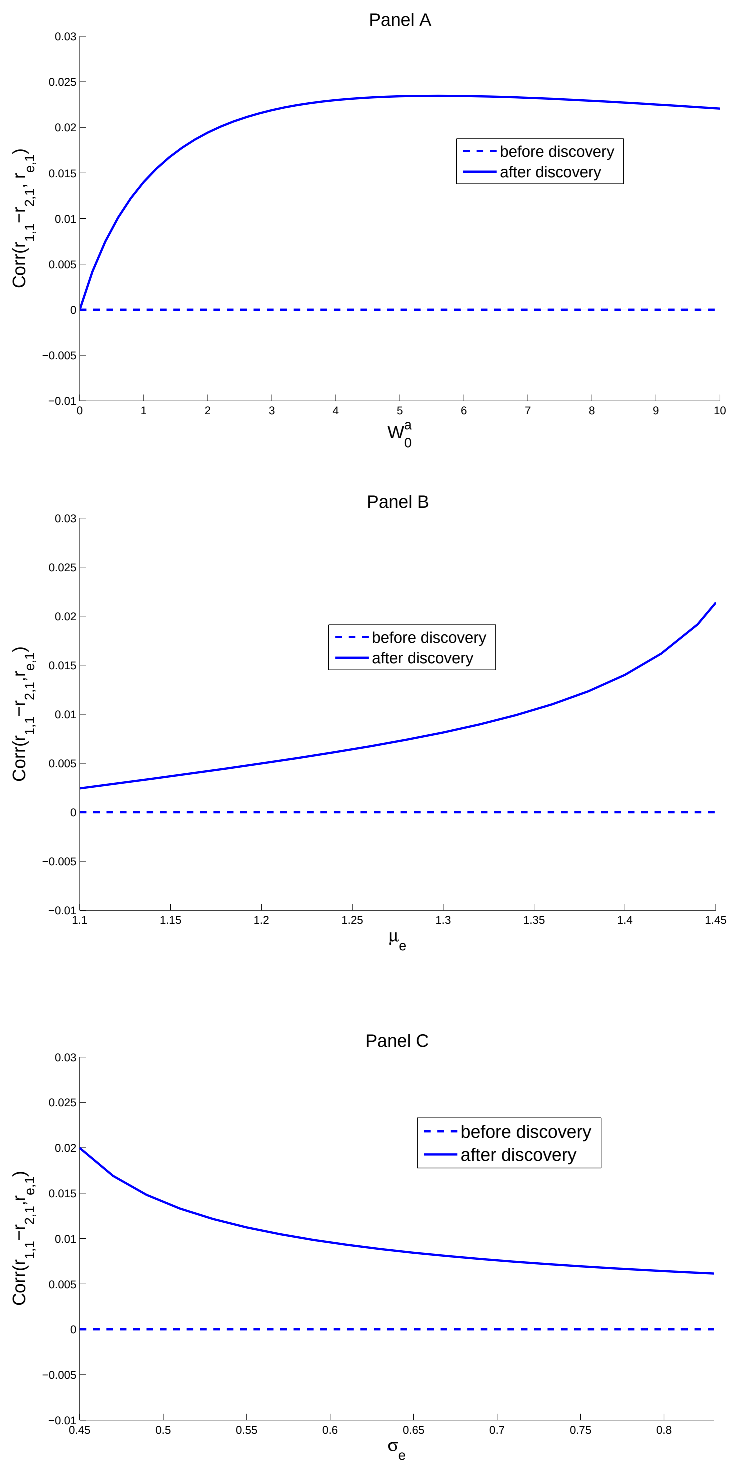
The first-order condition to the maximization problem (12) is given by (13). The first-order conditions for arbitrageurs are still given by (14) and (15). These optimality and market-clearing conditions lead to the results in the proposition.

Figure 1: **Anomaly Return**



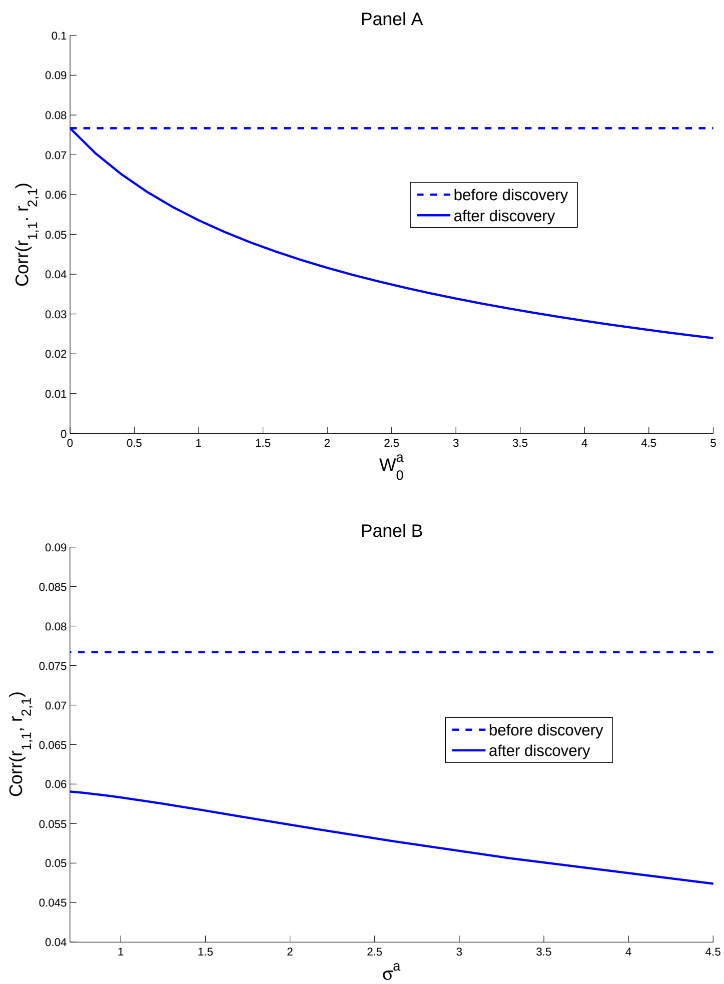
Panels A–C plot the expected anomaly return, $E[r_{1,1} - r_{2,1}]$, on arbitrageurs' initial wealth W_0^a, asset e's expected return μ_e and volatility σ_e, respectively. The parameter values are given by Table 1.

Figure 2: **Correlation Among Anomaly Returns**



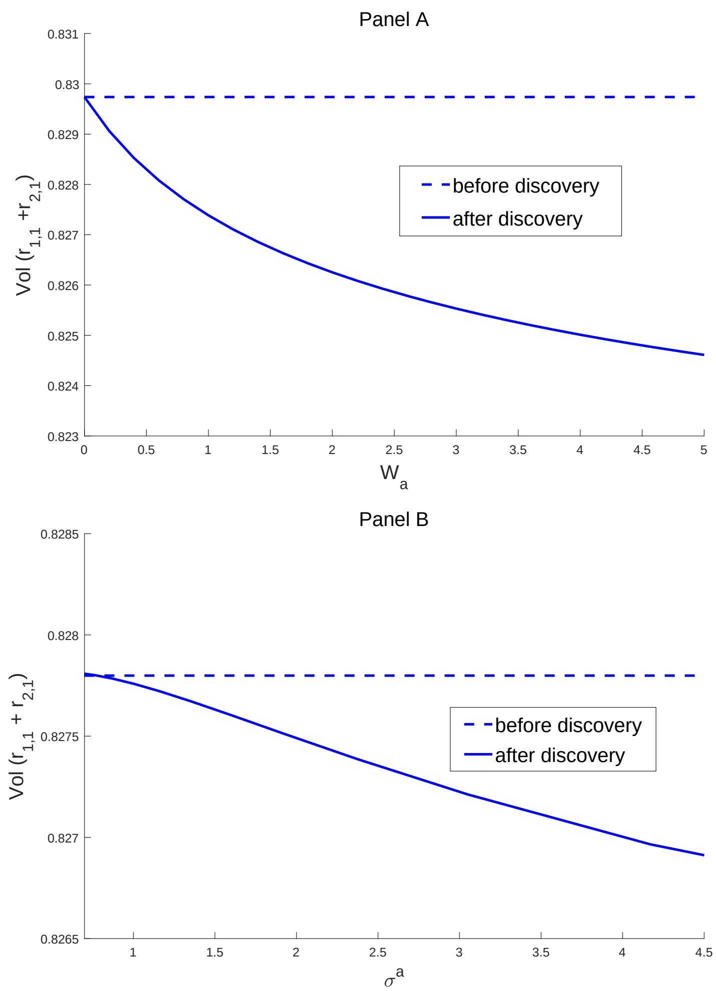
Panels A–C plot the correlation coefficient between the anomaly return and asset e's return, $Corr(r_{1,1} - r_{2,1}, r_{e,1})$, on arbitrageurs' initial wealth W_0^a, asset e's expected return μ_e, and its volatility σ_e, respectively. The parameter values are given by Table 1.

Figure 3: **Correlation Between Assets 1 and 2**



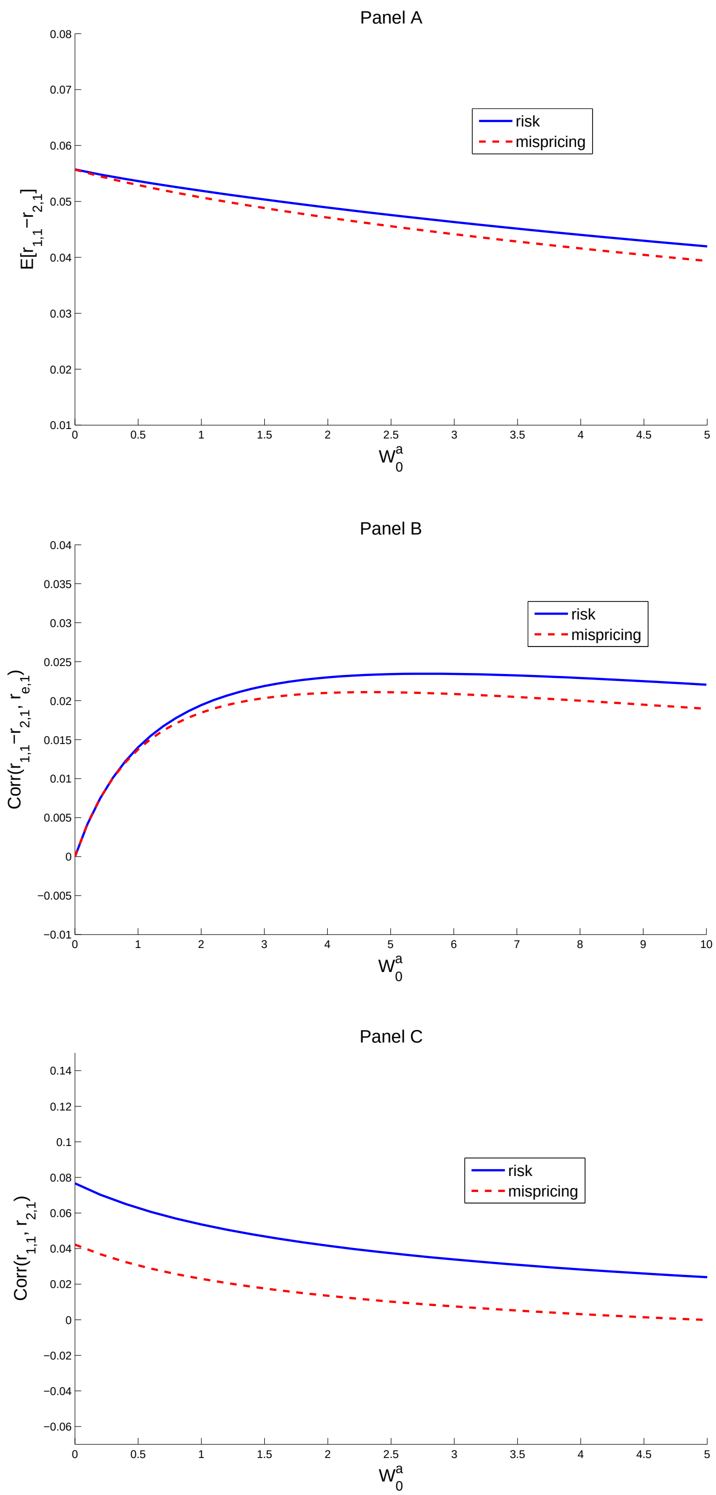
Panels A and B plot the correlation coefficient between assets 1 and 2, $Corr(r_{1,1}, r_{2,1})$, on arbitrageurs' initial wealth W_0^a, and their wealth volatility σ^a, respectively. Arbitrageurs' wealth volatility σ^a is an endogenous variable. We generate its variation by varying μ_e from 1.1 to 1.46. All other parameter values are given by Table 1.

Figure 4: **Diversification Benefits**



Panels A and B plot the volatility of the portfolio of Assets 1 and 2, $Vol(r_{1,1} + r_{2,1})$, against arbitrageurs' initial wealth W_0^a, and wealth volatility σ^a respectively. Arbitrageurs' wealth volatility σ^a is an endogenous variable. We generate its variation by varying μ_e from 1.1 to 1.46. All other parameter values are given by Table 1.

Figure 5: **Comparison: Asset Prices**



Panels A–C plot the expected anomaly return, $E[r_{1,1} - r_{2,1}]$, its correlation with asset e's return, $Corr(r_{1,1} - r_{2,1}, r_{e,1})$, and the correlation between assets 1 and 2, $Corr(r_{1,1}, r_{2,1})$, on arbitrageurs' initial wealth W_0^a, respectively. The solid line is for the risk-based case, and the dashed line the mispricing-based case. Parameter values: $b = 0.055$, and other parameter values are given by Table 1.

Table 1: Baseline Parameterizations

This table reports the baseline parameter values in our numerical analysis.

Parameter	W_0^a	k	ρ	μ	σ	p	μ_e	σ_e	p_e
Value	1	1	1	1.2	0.6	0.5	1.4	0.5	0.5

Table 2: Summary Statistics

This table reports the summary statistics. Mean $X_{i,t}$ and Std $X_{i,t}$ are the mean and standard deviation of the excess correlation, where the excess correlation $X_{i,t}$ is defined in (16). Mean hedge fund wealth volatility and Std hedge fund wealth volatility are the mean and standard deviation of the hedge fund wealth volatility. Hedge fund wealth volatility is the standard deviation of the monthly percentage changes in the assets under management by all U.S.-equity-focused hedge funds, and is estimated based on a rolling window of the previous 5 years. Mean Anomaly Trading is the average Anomaly Trading across all anomalies and all quarters. For each anomaly and each quarter, the Anomaly Trading is the average hedge fund purchase of decile 1 stocks minus that of decile 10 stocks.

Number of Anomalies	99
Number of Anomalies with t-statistic>1.5	86
Average Correlation among Anomalies	0.05
Mean Publication Year of the Anomalies	2000
Median Publication Year of the Anomalies	2001
Percentage of Anomalies based on Working Paper	10%
Mean Long-Short Monthly Anomaly Return	0.70%
Mean $X_{i,t}$	-0.10
Std $X_{i,t}$	0.14
Mean Hedge Fund Wealth Volatility	0.08
Std Hedge Fund Wealth Volatility	0.10
Mean Anomaly Trading	-0.15%

Table 3: Discovery and Correlation

This table reports the results from the panel regressions of the excess correlation $X_{i,t}$, which is defined in (16), on the dummy variable Discovery, which is 0 before the discovery time and 1 afterwards, Wealth_vol (t), which is the hedge fund wealth volatility normalized by its standard deviation, and the interaction term. Hedge fund wealth volatility is the standard deviation of the monthly percentage changes in the assets under management by all U.S.-equity-focused hedge funds, and is estimated based on a rolling window of the previous 5 years. The regressions are either equal-weighted or citation-weighted, and include anomaly fixed effects. The citation counts of an anomaly are its Google citation counts as of October 2016. Constant terms are omitted. T-statistics are reported in the parenthesis, and are based on standard errors that are clustered by anomaly. *, **, and *** indicate that the coefficients are statistically significant at the 10%, 5%, and 1% level, respectively.

Dependant Variable	Correlation (i,t)			
	Equal-Weighted	Citation-Weighted	Equal-Weighted	Citation-Weighted
Wealth_vol (t)			0.03***	-0.0096
			(7.01)	(-1.27)
Discovery × Wealth_vol (t)			**-0.05****	**-0.13*****
			(-2.37)	**(-3.00)**
Discovery	**-0.05*****	**-0.09*****	0.01	-0.063**
	(-6.39)	**(-9.11)**	(1.56)	(-2.05)
Anomaly Fixed Effects	Yes	Yes	Yes	Yes
Observations	59,358	59,358	31,469	31,469
R^2	0.04	0.10	0.06	0.18

Table 4: Discovery and Diversification Benefits

This table reports the results from the panel regressions. The dependent variable is Excess Volatility (i,t), which is the standard deviation, in month t, of monthly returns of the portfolio of deciles 1 and 10 of anomaly i minus the standard deviation of monthly market retunrs. The estimation is based on a rolling window of the previous 5 year data. The independent variables include the dummy variable Discovery, which is 0 before the discovery time and 1 afterwards, Wealth_vol (t), which is the hedge fund wealth volatility normalized by its standard deviation, and the interaction term. Hedge fund wealth volatility is the standard deviation of the monthly percentage changes in the assets under management by all U.S.-equity-focused hedge funds, and is estimated based on a rolling window of the previous 5 years. The regressions are either equal-weighted or citation-weighted, and include anomaly fixed effects. The citation counts of an anomaly are its Google citation counts as of October 2016. Constant terms are omitted. T-statistics are reported in the parenthesis, and are based on standard errors that are clustered by anomaly. *, **, and *** indicate that the coefficients are statistically significant at the 10%, 5%, and 1% level, respectively.

Dependant Variable	Excess Volatility (i,t)			
	Equal-Weighted	Citation-Weighted	Equal-Weighted	Citation-Weighted
Wealth_vol (t)			-0.001***	0.001***
			(-4.21)	(4.57)
Discovery × Wealth_vol (t)			**-0.004*** **	**-0.006*** **
			(-2.83)	**(-8.75)**
Discovery	**-0.007*** **	**-0.02*** **	0.002	-0.005
	(-2.98)	**(-2.94)**	(1.11)	(-0.66)
Anomaly Fixed Effects	Yes	Yes	Yes	Yes
Observations	59,358	59,358	32,240	32,240
R^2	0.01	0.10	0.06	0.05

Table 5: Discovery and Aribtrage Trading: Long Positions

This table reports the results from the panel regressions. The dependent variable is Anomaly Trading (i,q), which is the average hedge fund purchase of decile 1 stocks minus that of decile 10 stocks of anomaly i in quarter q. The independent variables include the dummy variable Discovery, which is 0 before the discovery time and 1 afterwards, Chwealth (q-1), which is the change of the aggregate asset under management of all the U.S.-equity-focused hedge funds in quarter q-1 normalized by its standard deviation, and the interaction term. The regressions are either equal-weighted or citation-weighted, and include anomaly fixed effects. The citation counts of an anomaly are its Google citation counts as of October 2016. Constant terms are omitted. T-statistics are reported in the parenthesis, and are based on standard errors that are clustered by anomaly. *, **, and *** indicate that the coefficients are statistically significant at the 10%, 5%, and 1% level, respectively.

Dependant Variable	Anomaly Trading (i,q)			
	Equal-Weighted	Citation-Weighted	Equal-Weighted	Citation-Weighted
Chwealth (q-1)			0.00	0.01
			(0.15)	(1.46)
Discovery × Chwealth (q-1)			**0.02*****	**0.02*****
			(2.81)	**(5.97)**
Discovery	**0.02****	**0.07*****	0.02*	0.06***
	(2.06)	**(7.58)**	(1.97)	(5.12)
Anomaly Trading (i,q-1)	-0.17***	-0.22***	-0.17***	-0.22***
	(-8.09)	(-2.75)	(-8.09)	(-2.73)
Anomaly Trading (i,q-2)	-0.14***	-0.27***	-0.14***	-0.27***
	(-5.18)	(-10.23)	(-5.20)	(-10.20)
Anomaly Fixed Effects	Yes	Yes	Yes	Yes
Observations	12606	12606	12420	12420
R^2	0.04	0.10	0.04	0.07

Table 6: Discovery and Aribtrage Trading: Short Positions

This table reports the results from the panel regressions. The dependent variable is Anomaly Trading (i,m), which is the average change in short interests of decile 10 stocks minus that of decile 1 stocks of anomaly i in month m. The independent variables include the dummy variable Discovery, which is 0 before the discovery time and 1 afterwards, Chwealth (m-1), which is the change of the aggregate asset under management of all the U.S. equity-focused hedge funds in month m-1 normalized by its standard deviation, and the interaction term. The regressions are either equal-weighted or citation-weighted, and include anomaly fixed effects. The citation counts of an anomaly are its Google citation counts as of October 2016. Constant terms are omitted. T-statistics are reported in the parenthesis, and are based on standard errors that are clustered by anomaly. *, **, and *** indicate that the coefficients are statistically significant at the 10%, 5%, and 1% level, respectively.

Dependant Variable	Anomaly Trading (i,m)			
	Equal-Weighted	Citation-Weighted	Equal-Weighted	Citation-Weighted
Chwealth (m-1)			-0.00	-0.01*
			(-0.38)	(-1.78)
Discovery × Chwealth (m-1)			**0.03****	**0.07*****
			(2.28)	**(3.36)**
Discovery	**0.04****	**0.04*****	0.05**	0.04***
	(2.15)	**(3.94)**	(2.56)	(3.65)
Anomaly Trading (i,m-1)	0.55***	0.64***	0.54***	0.62***
	(11.81)	(11.83)	(13.05)	(11.28)
Anomaly Trading (i,m-2)	-0.04	-0.08***	-0.03	-0.07***
	(-1.48)	(-3.24)	(-1.21)	(-2.78)
Anomaly Fixed Effects	Yes	Yes	Yes	Yes
Observations	9,791	9,791	9,791	9,791
R^2	0.27	0.35	0.27	0.36

Table A.1 Anomalies

This table provides the name, author, publication information of the 99 anomalies used in our paper. Notation: FAJ=Financial Analysts Journal; JAE= Journal of Accounting and Economics; JAR=Journal of Accounting Review; JBFA=Journal of Business, Finance & Accounting; JF=Journal of Finance; JEF=Journal of Empirical Finance; JFE=Journal of Financial Economics; JFM=Journal of Financial Markets; JFQA=Journal of Financial and Quantitative Analysis; JPE=Journal of Political Economy; RAS=Review of Accounting Studies; TAR=The Accounting Review; WP=Working Paper.

No.	**Anomaly Name**	**Author(s)**	**Date, Journal**
1	Beta	Fama & MacBeth	1973, JPE
2	Beta squared	Fama & MacBeth	1973, JPE
3	Earnings-to-price	Basu	1977, JF
4	O-score	Ohlson	1980, JAR
5	Dividends-to-price	Litzenberger & Ramaswamy	1982, JF
6	Unexpected quarterly earnings	Rendelman, Jones & Latane	1982, JFE
7	Change in forecasted annual EPS	Hawkins, Chamberlin & Daniel	1984, FAJ
8	36-month Reversal	De Bondt & Thaler	1985, JF
9	Forecasted growth in 5-year EPS	Bauman & Dowen	1988, FAJ
10	Leverage	Bhandari	1988, JF
11	% change in current ratio	Ou & Penman	1989, JAE
12	% change in quick ratio	Ou & Penman	1989, JAE
13	% change in sales-to-inventory	Ou & Penman	1989, JAE
14	Cash flow-to-debt	Ou & Penman	1989, JAE
15	Current ratio	Ou & Penman	1989, JAE
16	Quick ratio	Ou & Penman	1989, JAE
17	Sales-to-cash	Ou & Penman	1989, JAE
18	Sales-to-inventory	Ou & Penman	1989, JAE
19	Sales-to-receivables	Ou & Penman	1989, JAE
20	Amihud illiquidity	Amihud & Mendelson	1989, JF
21	Bid-ask spread	Amihud & Mendelson	1989, JF
22	12-month momentum	Jegadeesh	1990, JF
23	1-month reversal	Jegadeesh	1990, JF
24	6-month momentum	Jegadeesh & Titman	1990, JF
25	Net stock issue	Ritter	1991, JF
26	% change in depreciation-to-gross PP&E	Holthausen & Larcker	1992, JAE
27	Depreciation-to-gross PP&E	Holthausen & Larcker	1992, JAE
28	Book-to-market	Fama and French	1992, JF
29	Size (market cap)	Fama and French	1992, JF
30	Annual sales growth	Lakonishok, Shleifer & Vishny	1994, JF
31	Industry-adjusted change in employees	Asness, Porter & Stevens	1994, WP
32	New equity issue	Loughran, Ritter & Ritter	1995, JF
33	Sales-to-price	Barbee, Mukherji & Raines	1996, FAJ
34	Working capital accruals	Sloan	1996, TAR
35	Share turnover	Datar, Naik & Radcliffe	1998, JFM
36	% change in CAPEX - industry % change in CAPEX	Abarbanell & Bushee	1998, TAR

37	% change in gross margin - % change in sales	Abarbanell & Bushee	1998, TAR
38	% change in sales - % change in accounts receivable	Abarbanell & Bushee	1998, TAR
39	% change in sales - % change in inventory	Abarbanell & Bushee	1998, TAR
40	% change in sales - % change in SG&A	Abarbanell & Bushee	1998, TAR
41	# of consecutive earnings increases	Barth, Elliott & Finn	1999, JAR
42	Industry momentum	Moskowitz & Grinblatt	1999, JF
43	Financial statements score	Piotroski	2000, JAR
44	Industry-adjusted book-to-market	Asness, Porter & Stevens	2000, WP
45	Industry-adjusted cash flow-to-price ratio	Asness, Porter & Stevens	2000, WP
46	Industry-adjusted firm size	Asness, Porter & Stevens	2000, WP
47	Abnormal volume	Gervais, Kaniel, Mingelgrin	2001, JF
48	Dollar trading volume in month t-2	Chordia, Subrahmanyam & Anshuman	2001, JFE
49	Volatility of dollar trading volume	Chordia, Subrahmanyam & Anshuman	2001, JFE
50	Volatility of share turnover	Chordia, Subrahmanyam & Anshuman	2001, JFE
51	# of analysts covering stock	Elgers, Lo & Pfeiffer	2001, TAR
52	Scaled analyst forecast of one year ahead earnings	Elgers, Lo & Pfeiffer	2001, TAR
53	Dispersion in forecasted eps	Diether, Malloy & Scherbina	2002, JF
54	Changes in inventory	Thomas & Zhang	2002, RAS
55	Idiosyncratic return volatility	Ali, Hwang & Trombley	2003, JFE
56	Growth in long term net operating assets	Fairfield, Whisenant & Yohn	2003, TAR
57	Net operating assets	Hirshleifer et. al.	2004, JAE
58	RD_increase	Eberhart, Maxwell & Siddique	2004, JF
59	Investment to assets	Titman, Wei & Xie	2004, JFQA
60	Cash flow-to-price	Desai, Rajgopal & Venkatachalam	2004, TAR
61	Earnings volatility	Francis, LaFond, Olsson & Schipper	2004, TAR
62	Taxable income to book income	Lev & Nissim	2004, TAR
63	Change in common shareholder equity	Richardson, Sloan, Soliman & Tuna	2005, JAE
64	Change in long-term debt	Richardson, Sloan, Soliman & Tuna	2005, JAE
65	# of years since first Compustat coverage	Jiang, Lee & Zhang	2005, RAS
66	Financial statements score	Mohanram	2005, RAS
67	Price delay	Hou & Moskowitz	2005, RFS
68	R&D-to-market cap	Guo, Lev & Shi	2006, JBFA
69	R&D-to-sales	Guo, Lev & Shi	2006, JBFA
70	% change over two years in CAPEX	Anderson & Garcia-Feijoo	2006, JF
71	Composite equity issue	Daniel and Titman	2006, JF
72	Industry sales concentration	Hou & Robinson	2006, JF

73	Return volatility	Ang, Hodrick, Xing & Zhang	2006, JF
74	Return on assets	Fama and French	2006, JFE
75	Zero-trading days	Liu	2006, JFE
76	Abnormal volume in earnings announcement month	Lerman, Livnat & Mendenhall	2007, WP
77	Change in # analysts	Scherbina	2007, WP
78	Return on invested capital	Brown & Rowe	2007, WP
79	Asset growth	Cooper, Gulen & Schill	2008, JF
80	Financial distress	Campbell, et. al.	2008, JF
81	Industry-adjusted change in asset turnover	Soliman	2008, TAR
82	Industry-adjusted change in profit margin	Soliman	2008, TAR
83	3-day return around earnings announcement	Brandt, Kishore, Santa-Clara & Venkatachalam	2008, WP
84	Revenue surprise	Kama	2009, JBFA
85	Cash flow volatility	Huang	2009, JEF
86	Debt capacity-to-firm tangibility	Hahn & Lee	2009, JF
87	Cash productivity	Chandrashekar & Rao	2009, WP
88	Employee growth rate	Bazdresch, Belo & Lin	2009, WP
89	Real estate holdings	Tuzel	2010, RFS
90	Absolute accruals	Bandyopadhyay, Huang & Wirjanto	2010, WP
91	Accrual volatility	Bandyopadhyay, Huang & Wirjanto	2010, WP
92	Change in tax expense	Thomas & Zhang	2011, JAR
93	Maximum daily return in prior month	Bali, Cakici & Whitelaw	2011, JFE
94	Percent accruals	Hafzalla, Lundholm & Van Winkle	2011, TAR
95	Cash holdings	Palazzo	2012, JFE
96	Organizational capital	Eisfeldt & Papanikolaou	2013, JF
97	Asset Turnover	Novy-Marx	2013, JFE
98	Gross profitability	Novy-Marx	2013, JFE
99	Secured debt-to-total debt	Valta	2015, JFQA