A Spectroscopic Investigation of Excited States of the Nucleus 73Br

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A Spectroscopic Investigation of the Excited States of the Nucleus $^{73}$Br

A Thesis Presented in Partial Fulfillment of the Requirements for the Degree of
Master of Science

January 15, 2013

By
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Abstract

Gamma rays emitted in the de-excitation of the nucleus $^{73}$Br were created in two distinct experiments. Using Gammasphere and Microball, the first experiment selected nuclei created in the reaction for an incident $^{40}$Ca beam and a $^{40}$Ca target that were coincident with the creation of 1 $\alpha$ particle, 3 protons, and 4 gamma rays. These results were analyzed using a gating procedure that allowed for the validation of published level schemes and the identification of new rotational bands in the level scheme. Using Gammasphere, the Fragment Mass Analyzer, an ionization chamber, and neutron detectors, the second experiment selected nuclei created in the reaction of $^{36}$Ar with $^{40}$Ca that produced nuclei with $Z = 35$, $A = 73$, and no neutrons created. From this second set of data, Direct Correlations from Oriented Nuclei (DCO) Ratios were measured to identify the electromagnetic nature (E1, M1, E2) of the transitions between energy levels, both to verify spin states from published level schemes and to assign spins to newly observed states. This analysis revealed 5 new bands in the $^{73}$Br level scheme and determined, or restricted, the total angular momentum of 16 states. Finally, moment of inertia calculations revealed a correlation between positive parity in a band and larger nuclear deformation.
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$\text{DCO Ratio} \ (\gamma_1, \gamma_2) = (\gamma_1 \text{ at } \theta, \gamma_2 \text{ at } \pi/2) / (\gamma_1 \text{ at } \pi/2, \gamma_2 \text{ at } \theta)$, where the $\gamma$-ray cascade is $\gamma_1 \rightarrow \gamma_2$.

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**Figure 6.3:** Data from the so-called “C Band” of $^{73}$Br. The level energies in keV vs the quantity $J(J + 1)$, where $J$ is the total angular momentum of the corresponding level, are plotted. The linear relationship between the level energy and $J(J + 1)$ supports the characterization of this sequence of energy levels as belonging to a quantum mechanical rigid rotor.
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Chapter 1: Introduction

“In every particle of the world is a mirror. In each atom lies the blazing light of a thousand suns.”

Mahmud Shabestari, 15th century Sufi Mystic

The initial work in this thesis establishes the details about the gamma rays released during the de-excitation of $^{73}$Br. Although one might model the nucleus as spherical with uniform charge density, this is not the reality; the $^{73}$Br nucleus (containing 35 protons and 38 neutrons) deviates from the spherical model. In a nucleus with partially filled shells, the valence nucleons tend to polarize the core towards a deformed nuclear shape that can be characterized by a multipole expansion with the quadrupole term dominating. These quadrupole shapes can either be with axial symmetry (flattened = oblate, and elongated = prolate) or without axial symmetry (called a triaxial shape). The shape of a nucleus can vary with neutron or proton number, excitation energy, or angular momentum. These shape changes are caused by the rearrangement of protons and neutrons in the nucleus or by the response of the nucleus to rotation. Since, there is no way to observe these shapes directly with the human eye, an alternative method of identifying nuclear shapes will involve studying the pattern of the energies of gamma rays released along the various decay paths. In doing so, it will also be important to characterize the states according to their quantum numbers of angular momentum and parity. This thesis study will use the technique of determining ratios of Directional Correlations from Oriented nuclei (DCO ratios) in order to characterize the multipolarity of the electromagnetic radiation of the gamma rays, and thus infer the angular momentum and parity of the associated states.

There is no single model of the nucleus that universally explains all its properties, rather a few models that explain specific properties well. One of the most basic models of the atomic nucleus is the nuclear shell model, which was originally proposed in 1932 by Dmitry Ivanenko and further developed by three scientists
The basis of the nuclear shell model is similar to the atomic orbital model for electrons in that the nucleons are placed in energy levels with a unique set of quantum numbers. This model is illustrated in Figure 1.1. The Pauli Exclusion Principle must be satisfied by the arrangement of neutrons and protons. The shape of the nucleus is assumed to be spherical and this model predicts the existence of magic numbers of nucleons (2, 8, 20, 28, 50, 82, or 126 neutrons or protons) that are more tightly bound than the next higher or lower number of nucleons. A nuclear potential function accounts for the short range nuclear forces. Generally, this potential is something between the square well and the harmonic oscillator with an additional spin orbit coupling term. Frequently, a Woods-Saxon potential, which is a deviation from the harmonic oscillator potential is used. The spherical Woods-Saxon potential is written as:

$$V = - \frac{V_0}{1 + \exp \left( \frac{r - R}{a} \right)}$$

Equation 1.1

where $r$ is the distance of the nucleons with respect to the center of the nucleus. $R$ is the radius of the nucleus which is approximately equal to $1.1A^{1/3}$ fm and surface thickness $a$ is approximately equal to 0.5 fm and depth $V_0$ is approximately equal to 50 MeV.
Figure 1.1: The left diagram show the energy levels calculated with the shell-model potential. The right diagram shows the capacity and cumulative number of nucleons up to that level. The right side also shows the effect of the spin-orbit interaction which splits the levels with orbital angular momentum greater than zero into new levels.
The nuclear shell model predicts the following arrangement of protons and neutrons for the ground state of $^{73}$Br as shown in Table 1.1 below, where “x” indicates the presence of a proton or neutron in the given orbital:

**Table 1.1:** The quantum numbers for the shell model states are given as “$n\ l\ (j)$”, where $l$ is the orbital angular momentum value of the nucleon, $j$ is the total angular momentum of the nucleon, and $n$ is the number of times that a particular $l$ value has occurred. For instance, the “1s” state is the first, or lowest orbital with $l = 0$, while the “2s” state is the second lowest orbital with $l = 0$. The orbital angular momentum values are given by the traditional spectroscopic notation with $s$, $p$, $d$, $f$ corresponding to orbital angular momentum values of 0, 1, 2, 3.

<table>
<thead>
<tr>
<th>Energy Levels</th>
<th>Protons (35)</th>
<th>Neutrons (38)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1f(5/2) – capacity 6 (Highest Energy)</td>
<td>x</td>
<td>xxxx</td>
</tr>
<tr>
<td>2p(1/2) – capacity 2</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>2p(3/2) – capacity 4</td>
<td>xxxx</td>
<td>xxxx</td>
</tr>
<tr>
<td>1f(7/2) – capacity 8</td>
<td>xxxxxxx</td>
<td>xxxxxxxxxx</td>
</tr>
<tr>
<td>1d(3/2) – capacity 4</td>
<td>xxxx</td>
<td>xxxx</td>
</tr>
<tr>
<td>2s(1/2) – capacity 2</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>1d(5/2) – capacity 6</td>
<td>xxxxxxx</td>
<td>xxxxxxx</td>
</tr>
<tr>
<td>1p(1/2) – capacity 2</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>1p(3/2) – capacity 4</td>
<td>xxxx</td>
<td>xxxx</td>
</tr>
<tr>
<td>1s(1/2) – capacity 2 (Lowest Energy)</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

The intrinsic spin for a nucleon (neutron or proton) is $\frac{1}{2}$, so the possible values for the total angular momentum become $j = l \pm \frac{1}{2}$ where $l$ is the orbital angular momentum. It is known that protons and neutrons in the same nuclear orbital tend to pair to states of zero net angular momentum, so a nucleus with an odd proton or neutron will have a total angular momentum equal to the angular momentum of the last odd nucleon. In the case of $^{73}$Br, we would expect that the ground state would
have a spin and parity of \( j^\pi = 5/2^- \) because the odd proton is located in the \( 5/2 \) state with \( l = 3 \). The so-called “spin” of the nuclear state refers to the total angular momentum of that state, and the parity is the reflection symmetry property of the wave function of the state. The parity of a wave function is considered “positive” or “even” if the wave function remains the same when the spatial coordinates of the wave function are inverted. In Cartesian coordinates, this means that \( x \rightarrow -x, y \rightarrow -y, \) and \( z \rightarrow -z \). A simple 1-dimensional example of a function with positive parity is the cosine function. In the cosine function, \( \cos(-x) = \cos(x) \). Conversely, a wave function is considered “negative” or “odd” if the wave function only changes sign when the spatial coordinates of the wave function are inverted. A simple example of a function with negative parity is the sine function where \( \sin(-x) = -\sin(x) \). In three dimensions and in spherical coordinates, the parity of the wave function of a nuclear state is determined by the quantity \((-1)^l\); so the quantum state with \( j = 5/2 \) and \( l=3 \) would have \( \pi = (-1)^3 = -1 \), giving the predicted ground state for \(^{73}\text{Br} \) a negative parity. While it is true that one of the lowest states of \(^{73}\text{Br} \) is \( 5/2 \), the experimental data from both Heese and Plettner predict a level with an angular momentum of \( 1/2^- \) at 26.8 keV below this \( j = 5/2 \) energy level. This would indicate a change in the ordering of the levels from that predicted by the spherical shell model so that the ground state has one neutron in the \( 2p(1/2) \) orbital and two in the \( 1f(5/2) \) orbital. This changing in the order of filling of the shell model states may indicate that the ground state of \(^{73}\text{Br} \) is not spherical. The shell model generally uses a potential that has spherical symmetry and leads to the magic numbers, orbitals, and shells that are in my figure. When the nucleus loses its sphericity, the model is now called the Nilsson model. As a result, the orbitals move around relative to each other, and you often break the degeneracy and split a single shell into multiple shells of individual orbitals. So when you identify the location of the last proton or neutron, the shells have shifted around and you predict a different angular momentum for the last particle.
While the shell model is useful for describing how the behavior of individual nucleons affects the overall properties of the nucleus, particularly at low energies, the collective model of nuclear structure (collective because it considers the nucleons acting together) is often useful in accounting for the structure and behavior of nuclei in higher energy states. This collective model accounts for the fact that internal forces between particles inside the nucleus will differ from the forces on particles on the outer layer of the nucleus. This is analogous to the study of surface tension in water where water molecules within a liquid experience a slightly different set of forces than the water molecules at the surface of the water. This model, originally called the liquid drop model, was used to explain the behavior of nuclei in the fission reaction, but eventually to the approximation of the nucleus as possessing an average shape that is spherical and an instantaneous shape that is not. The collective model can be useful for nuclei that possess valence nucleons, or outer shells which are far from being filled. From our shell model analysis, the $^{73}$Br nucleus is one of these nuclei.

Accelerators at the Argonne National Laboratory are used to create nuclear reactions that produce nuclei at relatively high excitation energy and with very high angular momenta. These nuclei will lose a portion of their excitation energy and the vast majority of their angular momentum by emitting gamma rays as they cool down. The gamma rays are detected by Gammasphere, a device that can time stamp the gamma rays that are absorbed by the detectors. Their corresponding locations and energies are recorded in a supercomputer. When indexed with a software package, nuclear physicists can query the experimental data to identify which gamma emissions occurred simultaneously in time. With this information, scientists construct an energy level scheme from the data. Basically, this energy level scheme is a visual display of the numerous energy pathways for a nucleus undergoing gamma decay from highly excited states to the ground state. The first portion of this thesis is devoted to constructing such a level scheme for $^{73}$Br, and builds off of previous research from two research groups (Heese and Plettner).
Chapter 2: Background

This analysis of the $^{73}\text{Br}$ nucleus begins with consideration of the significant previous work of two different research groups led by Heese and Plettner$^1$. The energy level scheme determined by Heese is shown in Figure 2.1. Just as electrons are assigned discrete energy levels in the atom, nuclei also exist in discrete energy levels that obey the rules of quantum mechanics. Both the ground and excited states are displayed in the energy level diagram. The energies of excited states are unique to each isotope and result from the internal structure of the nuclei. Each energy level is labeled by its total energy in keV, its total angular momentum $J$ in half-integer units of $\hbar$ (since this nucleus has an odd number of nucleons), and its parity. An even parity is indicated by a + sign and an odd parity is indicated by a - sign. The energy diagram shown below in Figure 2.1 illustrates the pathways taken by the experimentally produced, highly excited $^{73}\text{Br}$ nuclei as they decay in energy and angular momentum toward the ground state.


According to Heese, “the nucleus $^{73}$Br possesses characteristics similar to other nearby (i.e., having neutron and proton numbers near 35-38) odd-$A$($A = N + Z$, the sum of the numbers of neutrons and protons) nuclei, as well as to other neutron deficient Br isotopes. Neutron deficient isotopes are generally less stable because they possess fewer neutrons in the nucleus than for more stable isotopes.” The
stable isotopes of Br are $^{79}$Br (35 protons and 43 neutrons, 50.7% natural abundance) and $^{81}$Br (35 protons, 46 neutrons, 49.3% natural abundance). “The neutron deficient Bromine isotopes display defining characteristics such as very large quadrupole deformations (indicating a reasonably large deviation from spherical shape), triaxiality, and the coexistence of different shapes (i.e., for a given excitation energy and angular momentum, there are two or more shapes which are highly probable for the nucleus to assume). Additionally, the most probable shape for the nucleus to take on can vary quite drastically for small variations in the particle number and/or angular momentum.” In his 1987 study, Heese addresses the contradictory and scarce information available about the $^{73}$Br nucleus and attempts to sort through the inconsistencies and establish a complete level scheme, including angular momentum and parity assignments and electromagnetic moments. Heese conducted gamma-gamma coincidence analysis on three different experiments to predict a level scheme. These experiments included studies of the reactions $^{40}$Ca($^{40}$Ca,α3p)$^{73}$Br at 155 MeV, and $^{40}$Ca($^{36}$Ar,3p)$^{73}$Br at 105 MeV and 125 MeV. Note that in this notation, the particles (or nuclei) from left to right are: (i) the target nucleus, (ii) the beam nucleus, (ii) any neutrons, protons and a particles emitted following the reaction, and (iv) the nucleus produced in the reaction. The set of particles listed on the right side within the parenthesis consists of those particles emitted as the compound nucleus (target + beam nuclei) cools down to produce the “final” nucleus, in this case,$^{73}$Br. Heese mentions that many peaks (gamma rays) above 800 keV are broadened with smaller intensities rendering it difficult to precisely pinpoint their values and angular distributions.

Heese also mentions that transitions below 100 keV were emitted isotropically and that he was unable to directly observe the lowest energy gamma present in his level scheme, the 27.3 keV gamma. In his detailed analysis, he acknowledges difficulty in making unique spin and parity assignments for lower level states, but clearly shows the emergence of three “rotational bands”. He makes comparisons to the $^{75}$Br nucleus to establish the spin and parity of some of the lower energy states.
The sequences of gamma rays observed in $^{73}\text{Br}$ (see Fig. 2.1) are good examples of a type of collective behavior exhibited by many nuclei. Figure 2.2 shows a comparison of collective (left side) excitations and non-collective (right side), or single-particle shell model excitations in two sample nuclei.

Figure 2.2: The two ways that a nucleus can generate angular momentum (spin). “Collective”, or classical behavior is shown to the left and “Noncollective”, or quantul, behavior is shown to the right. (Source: http://ns.ph.liv.ac.uk/mass110.html).
The collective excitations are well-described by a quantum-mechanical rigid rotor model. For example, if we begin with a value of classical rotational kinetic energy:

$$ E = \frac{1}{2} I \omega^2 $$

Equation 2.1

where $I$ is the moment of inertia of the rotating object, and $\omega$ is the angular frequency, then we can write the angular momentum as

$$ J = I \omega $$

Equation 2.2

By substitution, we develop the following expression

$$ E = \frac{J^2}{2I} $$

Equation 2.3

and we move to quantum mechanics and use this expression as our Hamiltonian operator. The quantum mechanical angular momentum operator is defined as:

$$ J^2 \mid j,m \rangle = \hbar^2 j(j+1) \mid j,m \rangle $$

Equation 2.4

Using this equation and assuming the energy is purely rotational, we can determine the energy using the eigenvalues that are:

$$ E = \frac{\hbar^2}{2I} j(j+1) $$

Equation 2.5

and the eigenstates are the spherical harmonics, $Y_{j}^{m}(\theta,\phi)$. If we graph the energy $E$ vs. $J(J+1)$ for a band, then the slope is equal to $\hbar^2/2I$, so we can extract the moment of inertia from the slope. Figure 2.3 shows such a graph for Band A of $^{73}$Br, which consists of the leftmost sequence of energy levels (from 286 keV to 8461 keV) from Figure 1. The linear relationship for the data clearly supports the characterization
of this set of energy levels as a rotational structure. For nuclei, this set of states is referred to as a rotational band and consists of a set of energy levels whose angular momenta differ successively by two units of angular momentum.

![A Band Level Energy vs. J(J+1)](image)

**Figure 2.3:** Data from the so-called “A Band” of $^{73}$Br, which consists of the leftmost sequence of energy levels (from 286 keV up to 8461 keV) from Fig. 1. The level energies in keV vs the quantity $J(J + 1)$, where $J$ is the total angular momentum of the corresponding level, are plotted. The linear relationship between the level energy and $J(J + 1)$ supports the characterization of this sequence of energy levels as belonging to a quantum mechanical rigid rotor.

Heese’s 1990 study expands upon the 1987 work where he established measurement of level energies and lifetimes, gamma ray angular distributions (intensity versus detection angle of individual gamma rays), and g-factors to determine magnetic moments. The spin and parity assignments made in the 1987 study, which were based off of gamma ray angular distributions, branching ratios, and level lifetimes, were largely confirmed, and the levels and bands extended to higher angular momentum and spin (45/2). Additionally, a 27 keV gamma ray which was postulated as the lowest level of the ground state in the 1987 work was observed and identified for the first time. In his previous 1987 study, angular
momentum assignments could not be made unambiguously because the ground state spin was unknown and the level scheme at lower energies was complex. However, in this 1990 study, the multipolarity of several transitions of $^{73}$Br were identified, allowing spin and parity assignments to be made. A discrete state for an atomic nucleus is characterized by its parity, spin, and energy level. To deduce this information about a state, we study the gamma emission for transitions between states. The gamma emission will have an energy corresponding to the difference between its initial state and its final state. The multipolarity of the photon corresponds to the angular momentum that the gamma ray carries away from the nucleus. Basically, if we know something about the spin and parity of the initial state, we can deduce the spin and parity of the final state by studying the multipolarity of the emitted gamma ray.

Most notably, this work identifies the ground state angular momentum of the level scheme as $1/2^-$, which differs from the $3/2^-$ suggested by the 1987 Heese paper and the value $5/2^-$ predicted by the nuclear shell model, as mentioned in Chapter 1 of this thesis.

A research group led by Plettner also analyzed $^{73}$Br, but focused on studying this nuclei at very high rotational frequencies and angular momenta, corresponding to highly excited states. Plettner suggests some characteristics for nuclei with atomic masses lying between 70 and 80. She describes a tendency of these nuclei to compete between prolate and oblate configurations at low spin states and the existence of a shell gap at near prolate and oblate shapes for higher spin states. Data was obtained from an experiment that employed the reaction $^{40}$Ca($^{40}$Ca,α3p)$^{73}$Br, using a 185 MeV beam from the XTU tandem accelerator at Laboratori Nazionali di Legnaro in Padova, Italy. Events with triple gamma coincidences (three gamma rays emitted simultaneously in time and produced in the same nuclear reaction) were recorded and the Radware (Radford, 1995) computer package was used for DCO analysis of the events. On the basis of the DCO ratios, Plettner confirmed all the spin
and parity states of known transitions and added an additional band D to the energy level diagram for \(^{73}\text{Br}\), shown in the figure below. Bands A, B and C are the three rotational bands originally identified by Heese, and each band was extended significantly to higher angular momentum states in the Plettner study. The resulting level scheme from Plettner’s research is shown in Figure 2.4.

\[ \text{Figure 2.4: Plettner’s Proposed Level Scheme for }^{73}\text{Br} \text{ from Physical Review C 62 (2000)} \]
The primary focus of Plettner’s work was to identify band termination for any of the bands in $^{73}\text{Br}$. This termination of rotational bands occurs when a configuration manifests itself as a collective rotational band at low spin values, but loses its collectivity with increasing spin. In a rotational band, the nucleus rotates about an axis perpendicular to the nuclear axis of symmetry.

Physically, the nucleus changes shape gradually from a collective (near prolate) shape into a noncollective oblate, prolate, or spherical shape. Plettner was able to follow one of the four bands to termination. The individual neutrons and protons contribute an angular momentum composed of their intrinsic spin plus their orbital angular momentum. Once the angular momentum of each individual particle is exhausted, then the band terminates. Theoretically, the angular momenta of the individual nucleons expected to contribute to the total angular momentum of the nuclear states of a particular band is known. Therefore, it is possible to predict the maximum value of total angular momentum the nucleus can have in that band. This maximum value is the value of angular momentum at which the band should terminate. Experimentally, the intensities of the gamma ray transitions in a band will smoothly decrease the higher up the band one looks. When the data are of sufficient quality, missing (or maybe absent) transitions at the top of the band can be taken as evidence for the termination of a band.
Chapter 3: Experimental Procedure

Our experimental analysis begins by verifying the energy diagrams suggested by Heese and Plettner and moves towards adding new bands and information to these established energy level diagrams. Three experimental devices, Gammasphere, Microball, and the Fragment Mass Analyzer were essential to providing the data for studying $^{73}$Br.

The Gammasphere array (Lee, 1990) is a spherical shell with 110 high volume, high purity germanium detectors (surrounded by Bismuth Germanate (BGO) Compton Suppression Shields (shown in Figures 3.1 and 3.2). It is designed to study nuclei at extreme conditions (high angular momenta, high isotopic spin, high excitation energy) by studying the gamma rays emitted from these nuclei.

Figure 3.1: Gammasphere array and its component parts.
(Source: http://www.symmetrymagazine.org/cms/?pid=1000021)
Figure 3.2: (left) High volume, high purity germanium detectors surrounded by Bismuth Germanate (BGO) Compton Suppression Shields(right) Diagram of germanium detector showing electronics box(1), BGO Crystal(2), and Hyper-pure Germanium(3)
(Source: http://www.physics.oregonstate.edu/~kranek/research1.htm).

The device known as Microball (shown in Figure 3.3) consists of 95 CsI(Tl) scintillators closely packed to cover 97% of the 4π steradians (Sarantites,1996). It fits inside of Gammasphere’s target chamber. This device is designed to detect low mass particles (Hydrogen-1, Deuterium, Tritium, Helium-3, and Helium-4). Microball enhances the resolving power of Gammasphere by selecting specific charged particle channels among a large number of possible pathways. For instance, the beam nucleus and the target nucleus typically combine to produce a so-called compound nucleus. That compound nucleus may sometimes emit two protons and one alpha particle, other times emit three protons, and still other times emit one proton and two alpha particles. These combinations of emitted particles are called the “particle channels.” The Microball detector can identify and differentiate between these different particle channels, making it possible to properly correlate the gamma rays detected in Gammasphere with the specific final nucleus produced in the reaction. Microball can also determine the direction of the recoiling target nucleus and its momentum.
The Fragment Mass Analyzer (FMA) is a triple-focusing recoil mass spectrometer, designed to separate heavy nuclei produced in nuclear reactions and disperse them in mass per unit charge onto the focal plane (Davids, 1989). When Gammasphere is installed at the target position of the FMA, the FMA can provide a clear signal of the atomic mass ($A$) of each nucleus that is produced in the reaction, and this information is associated with the gamma rays detected in Gammasphere. The FMA also works in tandem with an ionization chamber, which identifies the atomic number ($Z$) of the nuclei being produced. This allows Gammasphere to construct data sets for specific nuclei essentially free from the background noise of other nuclei.

There were two experiments that provided the data for the current study of the $^{73}$Br nucleus. The first experiment, referred to as GSFMA19, was conducted at Argonne in 1998 using both the Gammasphere and Microball detectors. In this experiment, a reaction between a $^{40}$Ca beam and a $^{40}$Ca target took place in the target chamber,
and several nuclei were created, including $^{73}\text{Br}$. Events which produced 1 $\alpha$ particle and 3 protons in the Microball detector coincident with 4 gamma ray emissions in Gammasphere were marked as events associated with $^{73}\text{Br}$ nuclei. The quadruple coincidence requirement and the high kinetic energy of the incident beam biased the data towards nuclei produced in states with high excitation energy and large angular momentum. The data obtained in this experiment were used to confirm an energy level scheme for $^{73}\text{Br}$ based on results of both a DePaul analysis as well as research conducted by Plettner and Heese. In verifying the energy level scheme, a special emphasis was placed on studying the lower energy states.

The second set of data, referred to as GSFMA123, was produced by a beam of $^{36}\text{Ar}$ incident on a $^{40}\text{Ca}$ target. Particles created in the target chamber were analyzed by the FMA, ionization chamber, and 26 neutron detectors. Gamma rays were again detected by Gammasphere. The experiment was originally intended to study $^{74}\text{Rb}$ nuclei, but produced $^{73}\text{Br}$ as its strongest channel. An event was classified as having created a $^{73}\text{Br}$ nuclei if there was no signal in the neutron detector, the ionization chamber showed $Z = 35$, and the FMA showed $A = 73$. There was no specific gamma coincidence requirement placed on this data, and the data was biased towards measuring nuclei at fairly low excitation energy. The GSFMA123 data was used primarily for a Direct Correlations for Oriented Nuclei (DCO) analysis to make spin and parity assignments for newly observed excited states and, where possible, to better understand the shape of $^{73}\text{Br}$ nuclei in its different configurations of neutrons and protons.
Chapter 4: Data Analysis Procedure

Two critical techniques were used in this analysis: double gating and DCO analysis of gamma rays.

4.1 Double Gate Coincidence and Establishment of a Level Scheme
The double gate coincidence analysis begins with the GSFMA19 data set containing gamma rays released from $^{73}$Br nuclei. The raw data are organized by “channels”, which are linearly proportional and easily calibrated to their corresponding gamma ray energies. This data is then sorted into a three dimensional cube with gamma ray energy on each axis. Borrowing a visual from Derek Svelnys’s thesis, the cube can be visualized as:

![Figure 4.1: Representation of an event in a data cube.](image)

If three gammas $(E_{\gamma_1}, E_{\gamma_2}, E_{\gamma_3})$ of an “event” are recorded simultaneously in time, then a count is recorded at this location in the cube. The more times a triple coincidence of the three gamma occurs, the more counts are recorded. A program called levit8r from the Radware software package (Radford, 1995) is used to identify gamma rays that are in time coincidence with two other gamma rays believed to be
part of a band. The requirement that the gamma rays are observed simultaneously in time guarantees that they each resulted from the creation of a particular $^{73}$Br nucleus corresponding to a single nuclear reaction event. The gamma-ray coincidence data can be used to confirm, or extend previously observed gamma-ray sequences.

For example, Band A of the $^{73}$Br energy diagram contains a gamma-ray series with energies of 2030-1834-1653-1470-1317-1165-995-804-583-187.5 keV, according to Heese (see Figure 2.1) If two gamma rays, with energies 1834 keV and 1653 keV, are selected for the double gate, all gamma rays coincident (within 20ns) with these two gamma rays will be observed in the data set. This is shown in Figure 4.2 below obtained from levit8r:

![Figure 4.2: Gamma Rays coincident with a double gate on 1834 keV and 1653 keV from Band A.](image)

If the Heese level scheme (Figure 2.1) is correct, we expect peaks at 2030, 1470, 1317, 1165, 995, 804, 583, and 187.5 keV. If two gamma rays are chosen for the double gate that don’t exist as part of a band, the resulting spectroscopic diagram will not show any peaks. For example, the 804 keV gamma ray and the 735 keV gamma ray exist in two different bands, so a double gate around these two peaks should reveal no clear resulting peaks, as shown in Figure 4.3 below, where no observable peaks can be seen.
This technique proved extremely useful in verifying the energy level diagrams established by Heese and Plettner, as well as considering additional bands present in the DePaul analysis. Details and examples of this analysis are included in Chapter 5, section 1.

4.2 DCO Analysis and Confirmation of Unknown Angular Momentum States

An analysis tool called the Directed Correlations of gamma rays from Oriented states (DCO) is the second critical analysis tool that can be used to examine the nature of many of the gamma-ray transitions between energy levels and to assign spins and parities for to the newly observed states (Krane, 1973). The electromagnetic (gamma) radiation from a decaying final nucleus is mostly of dipole or quadrupole nature or a mixture of both of these types of radiation (see Kraemer – Flecken and Krane). Knowledge about the multipole nature of the gamma rays emitted allows us to establish the angular momentum and parity of the initial and final states, or at least to restrict the allowed possible values of these quantum numbers for particular states. When a nucleus is created in a reaction, the angular momentum vector of the excited nuclear state is aligned, or preferentially oriented, in a plane that is perpendicular to the direction of the incident beam. Because of this, gamma rays emitted from these excited states will have intensity patterns that vary as a function of angle relative to the beam axis; i.e., the angular distributions of gamma rays produced in nuclear reactions are non-isotropic. The shape of the angular distribution for a specific gamma ray depends on the multipolarity (L=1, 2, etc.) of that particular transition. A radiation strength symmetry is noted between
radiations that occur at 0 degrees and 180 degrees and with those occurring at 90 degrees and 270 degrees. (See Ejiri and de Voigt)

These ideas can be extended for a DCO analysis in which the intensities of pairs of gamma rays known to be coincident in a level scheme are measured, and ratios are obtained when the intensities of the pairs of gamma rays are detected at two different sets of angles relative to the direction of the incident beam. For example, let’s consider the intensity $I_1$ of a chosen gamma ray, say that of the 1834 keV transition, that is detected by a forward (or backward) detector, 0° (or 180°) relative to the incident beam, and in coincidence with another transition, say the 1653 keV gamma ray, detected by a side detector (90°). Next consider the intensity $I_2$ when the angles of detection for the two gamma rays are reversed: The 1834 keV gamma ray is now detected by a side detector in coincidence with the 1653 keV gamma ray detected by a forward detector. The ratio of $I_1$ to $I_2$ represents a DCO ratio for this pair of coincident gamma rays, and can be used to identify the types of transitions occurring, in particular, the multipolarities ($L = 1$ or 2) of the two transitions. In our example, the DCO ratio can be represented as:

$$
DCO = \frac{I_1}{I_2} = \frac{\text{Intensity of 1834 keV (forward) gated on 1653 keV at (side)}}{\text{Intensity of 1834 keV (side) gated on 1653 keV at (forward)}}
$$

or more generally:

$$
DCO = \frac{I_1}{I_2} = \frac{\text{Intensity of Gamma 1 (0°, 180°) gated by Gamma 2 (90°)}}{\text{Intensity of Gamma 1 (90°) gated by Gamma 2 (0°, 180°)}}
$$

The exact value and definition of the DCO ratio can vary depending on the physical placement, or geometry, of the detectors used in a particular experiment, and on the
specific sets of detectors selected for use in the analysis. For example, in the DCO analysis used by Plettner (Plettner 2000), DCO ratios of 0.5 corresponded to a gamma-gamma cascade in which one transition was a quadrupole (L=2) and the other was a dipole (L=1), and DCO ratios of 1 corresponded to a pair of quadrupole (L=2) transitions. Specifically Plettner used this analysis to confirm the E2 (electric quadrupole) character for the 1651 keV gamma ray in Band A, for the 1471 keV, 1637 KeV, 1780 keV gamma rays in Band B, for the 462 keV and 1593 keV gamma rays in Band C, and for the 1210 keV gamma ray in Band D.
Chapter 5: Data Analysis and Results

5.1 Establishing and Validating the Level Scheme

This analysis begins by examining band A of the level scheme proposed by Heese in 1990 and validating it by using the data obtained in the DePaul Analysis. Heese’s energy level diagram shows three bands, which we will call A, B, and C (following the labeling scheme used by Plettner) where A is the band furthest to the left and C is the band furthest to the right, shown previously in Figure 2.1.

We will also consider the energy level diagram contributed by Plettner (2000) which is shown previously in Figure 2.3. This diagram proposes an additional Band D and includes higher energy states than the one proposed by Heese. Our validation of the energy level schemes proposed above uses a data set known as GSFMA19 which is stored on the server at DePaul. These gammas and their time stamps represent the data used to produce the following histograms.

**Band A**

Beginning with band A, a double gate isolating the 1834 and 1653 gamma rays, which are a part of the proposed level scheme, produces several intense peaks at 176, 186, 214, 258, 583, 804, 994, 1168, 1316, 1469 keV. (see Figure 5.1). This seems to agree very well with Heese’s suggested band A with transitions of energies 583-804-995-1165-1317-1470-1653-1834 keV. Since the gating occurred for the 1834 and 1653 keV gamma rays, these do not appear in the figure.
Additionally, there are two broader peaks observed at 1963 and 2108 keV. These do not seem to match the band A suggested by Heese, but match the 1959 and 2100 keV gammas shown in Plettner’s band A. Four gamma rays suggested by Heese and Plettner which do not appear on our histogram are of energies 45, 63, 108, and 27 keV.

The next gate is a double gate around the 186.4 and 583 keV gamma rays near the bottom of the band. The overall results are shown below in Figure 5.2 and the peaks at 1468, 1317, 1168, 995, 804, 258, 212, and 176 keV are clearly visible.

Figure 5.1: Double gate on 1834 keV and 1653 keV using levit8r.

Figure 5.2: Double gate on 186.4 keV and 583 keV using levit8r.
However, a closer look at the lower level energy spectrum reveals the presence of the 46, 62, 108, and 152 keV gamma rays that did not show up in the double gate around the 1834 and 1653 keV transitions.

Next, we gated around the 1829 keV and 1964 keV gamma to look for higher energy gamma rays suggested by Plettner’s energy level scheme. In Figure 5.3, the presence of a gamma ray transition at 2302 keV is indicated, although there is a large uncertainty in its peak value. No gamma rays above this 2302 keV level are observed, so the final DePaul level scheme does not include some of Plettner’s higher energy gammas.

![Figure 5.3: Double gate on 1829 keV and 1964 keV.](image)

Band A consists of the sequence of gamma rays (from low to high energy states) of energies 27(not observed)-258-108-186-583-804-994-1168-1316-1469-1653-
1834-(1963)-(2108) or 27-258-62-46-186-583.4-804-994-1168-1316-1468-1654-1834-(1963)-(2108)-(2302) keV where the 1963, 2108, and 2302 keV gamma ray energies are observed with large uncertainties in the energy. The 27 keV gamma does not appear in the histogram with any of the gating procedures. This isn’t surprising because detector efficiencies are smaller at energies less than 100 keV.

**Band B**

Our analysis now moves to the band B suggested by Heese to have transitions of 574-735-885-1036-1181-1312-1476-1643-1784 keV. Let’s begin with a double gate around 885 and 1036 keV. The resulting spectrum, Figure 5.5 is shown below.

![Figure 5.5: Double gate on 885 keV and 1036 keV using levit8r.](image.png)

This histogram reveals strong peaks coincident with the 885 and 1036 keV transitions in band B. The higher energy gammas suggested by this figure are 503, 574, 654, 735, 1182, 1313, 1473, and 1640 keV. All of these peaks appear in the Heese and Plettner level schemes and are included in the DePaul analysis. A double gate on 1474 and 1313 keV, while examining higher energy peaks (Figure 5.6) reveals the presence of additional higher order peaks indicated by the Plettner level scheme and the DePaul Analysis. These additional peaks occur at 1782-1998-2252 keV. The highest energy gamma suggested in the earlier DePaul analysis at 2586 keV cannot be verified by this analysis.
Figure 5.6: Double gate on 1474 keV and 1313 keV using levit8r with zoom on higher energy gammas.

Band C
Moving to band C, we begin with a double gate around the 1465 and 1596 keV gamma rays. This results in the histogram shown below in Figure 5.7.

Figure 5.7: Double gate on 1465 keV and 1596 keV using levit8r.

The following peaks are identified in this analysis: 178, 305, 455, 462, 719, 850, 918, 953, 1070, 1216, 1348, 1805, 2065, and a small, broadened peak at 2360 keV. This analysis validates the band C presented by Heese, Plettner, and the DePaul analysis.

Band C splits at the lower energy levels into three different decay paths. Path C-1 contains transitions of 305, 151 and 461 keV. To verify its existence and placement in band C, we double gate around the 305 and 461 keV gamma rays (shown in
Figure 5.8) and observe strong peaks at 151, 178, 719, 850, 1072, 1218, and 1348 keV. Peaks that are missing are of energies 455 keV and 916 keV, indicating that these gamma emissions are not part of this decay path. The resulting histogram is shown below.

![Figure 5.8: Resulting histogram from double gating around 305 and 461 keV.](image)

Path C-2 contains transitions of 461 and 455 keV. Performing a similar double gate around these two gamma rays (see Figure 5.9) results in all the higher energy gamma rays of band C, and excludes the gamma rays that are part of either Path C-1 or C-3.
Finally Path C-3 contains a 918 keV gamma ray leading directly into a gamma ray of energy 719 keV. A double gate around the 918 and 719 keV transitions (Figure 5.10) shows that this is also part of band C because it contains all the higher energy gamma rays of this band, but excludes the gammas from Paths C-1 and C-2.

Figure 5.9: Resulting histogram from double gating around 455 and 461 keV.

Figure 5.10: Resulting histogram from double gating around 918 and 719 keV.
Band D
The final band suggested by Plettner, but not identified by Heese is Band D, which branches off of Band A and contains gamma rays of energies 1210-1573-1608-1757-1786-1875 keV. A double gate around the 1573 and 1210 keV transitions reveals peaks at 186, 583, 805, 991, 1610 keV, and a broadened peak at 1761 keV. The resulting histogram is shown below in Figure 5.11. Peaks with energies 186, 583, and 805, and 991 keV are part of band A. Note that no peak from the band A above 991 keV appears. This verifies the branching of band D off of the 2855 keV energy level of band A.

![Histogram of Band D](image)

**Figure 5.11:** Resulting histogram from double gating around 1573 and 1210 keV.

The DePaul analysis examines a gate around higher energy gammas (1573 and 1611 keV) to pinpoint a few additional higher energy gamma rays at 1760 keV, 1788 keV, and 1878 keV, as shown below in Figure 5.12.

![Zoomed into higher energy channels](image)

**Figure 5.12:** Resulting Histogram for double gating around 1573 and 1611 keV zoomed into higher energy channels.
Crossover Gammas Between Band D and Band A

Some nuclei in our experiments do not decay directly down band D, but rather they crossover into the band A at higher energy levels. Our analysis predicts the following crossover transitions between band D and band A (see the adjacent Figure 5.13).

![Crossover transitions between band D (right) and band A (left).]

The energy of a crossover gamma ray is obtained by subtracting the value of the energy level in Band A from a nearby, higher energy level in Band D. For example, a crossover gamma ray of energy 2205 keV could exist coming from the 9011 keV energy level in Band D and dropping to the 6806 keV energy level in Band A. The evidence for these crossover transitions exists when a double gate is taken that includes a higher energy gamma ray from band D and a region that includes the energy of a possible crossover gamma ray. For example, the spectrum from a double gate around the 1788 keV gamma from band D and the 2205 keV (see Figure
5.14) crossover gamma from the band A displays all the gammas lower than 1468 keV of band A. This validates the existence of a crossover gamma ray of 2205 keV.

![Figure 5.14: Double Gate around the 2205 keV gamma and the 1788 keV gamma.](image)

Consider an additional example of the crossover gamma ray at 1209 keV from the 4065 keV energy level of band D to the 2856 keV energy level of band A. We gated this 1209 keV crossover gamma with the 1575 keV gamma from the band D and identified all the gamma rays below 2856 keV in band A and many of the gamma rays above 4065 keV in band D. This is shown in Figure 5.15 below.

![Figure 5.15: Double gate around the 1575 keV and 1209 keV gamma.](image)

The additional crossover gammas between band D and band A (1913 keV and 1618 keV) were located in a similar way using the double gating procedure described above.

Finally, band D indicates the presence of a decay from a single angular momentum state (tentatively 41/2+) into the 9011 keV energy level of band D. The gamma ray
emitted in this decay has an energy of 1835 keV. If we double gate around the 1835 keV gamma and the 1760 keV gamma in band D (see below Figure 5.16), we verify the existence of this higher energy state because the lower energy band A and band D gamma rays emerge in the histogram.

![Figure 5.16: Double gate around the 1835 keV and 1760 keV gamma.](image)

New Bands (E and F)

A previous DePaul analysis suggested the possibility of a band branching off band A from energy level at 474 keV. The first gamma ray above this level has an energy of 525 keV and we performed a double gate with a low energy gamma ray from band A (186 keV) and the 525 keV transition to see this new band, Band E appear. (See Figure 5.17)

![Figure 5.17: Double Gating around 525 keV and 186 keV gamma.](image)
Interestingly, there wasn’t just one band, but 2 bands emerging from band A, and we called them Band E and Band F. The spectrum from the double gate around gamma rays with energies of 186 keV and 525 keV is shown below. There are well-defined peaks at 108, 176, 258, 522, 585, 805, 819, 904, 962, 978, 995, 1032, 1112, 1166, and 1316 keV. This becomes challenging because there appear to be multiple crossover gammas between band E and band A, and some of these peaks belong to band F and some to band E.

A closer look at peaks coincident with 525 and 817 keV gamma rays (see figure 5.18) reveals the existence of the following band which we’ll call band E: 186-525-904-819-960-1112-1250 keV.

![Figure 5.18: Double gate around 525 keV gamma and 819 keV gamma revealing peaks in band E.](image)

Closer analysis reveals seven crossover gamma rays which decay from this newly proposed E band into the ground state band A (see 2022, 772, 1938, 1821, 1665, 846, 713 keV gammas). As an example, let’s consider the 846 keV gamma ray. From the 1903 keV energy level of band E, an 846 keV gamma ray is emitted and the
nucleus decays to the 1057 keV level of band A. A double gate around this 846 keV gamma ray and the 583 keV gamma ray from the ground state band reveals several peaks (817, 958, 1112 keV) from band E, as shown below in Figure 5.19. A similar analysis was performed with the additional six crossover gammas between Band E and Band A to verify their existence.

![Figure 5.19: Double gate around 846 keV crossover gamma and 583 keV gamma in Band A reveals many peaks in band E.](image)

There also appears to be another band coincident with the 525 and 186 keV gamma rays which is independent of band E and we will call this band F. This band contains a doublet of the 525 keV gamma ray, meaning that there are two coincident gamma rays with nearly identical energies. A double gate around the two 525 gamma rays reveals the following peak energies, as shown in the histogram below (see figure 5.20): 525, 525, 901, 976, 1107, 1264, 1375, and 1533 keV. These gamma rays will establish band F, which like band E, has several crossover gamma rays that decay into band A.
Figure 5.20: Double Gate around 525 keV and 525 keV gammas revealing band F.

From this band F, there are five crossover gamma rays which decay into band A: 1750 keV, 1652 keV, 1540 keV, 1368 keV, 1050 keV gammas. The first gamma ray, with an energy of 1750 keV, decays from the 5775 keV level of band F into the 4021 keV level of band A. The existence of this crossover gamma ray is verified by a double gate around the 1750 and 1166 keV transitions, as shown below in Figure 5.21. The presence of the 804, 995, and 1165 keV gamma rays from band A and the 1375 keV gamma ray from band F indicate that the link between these two bands is the 1750 keV transition.
Figure 5.21: Double gate around the 1750 and 1166 keV transitions reveals higher energy peaks from band F and lower energy peaks from the band A, verifying that 1750 keV is a crossover gamma from band F to band A.

An additional crossover gamma between band F and the ground state band A is the 1652 keV gamma ray emitted from the 4508 keV energy level of band F and decaying into the 2856 keV level of the ground state band A. A double gate around the 1264 keV gamma ray of the band F and the 995 keV gamma ray of the ground state band A reveals the higher energy transitions belonging to band F (e.g. at 1374 keV) and lower energy transitions from the band A (e.g. at 995, 803, and 583 keV) as shown in Figure 5.22 below. The additional crossover gammas between band F and band A were verified using a similar double gating procedure.
The DePaul analysis builds upon the work of Heese and Plettner by establishing the additional bands E and F in the level scheme and the crossover gammas between these bands and Band A, as shown in the level scheme below (Figure 5.23).

**Figure 5.22:** Double Gate around 1264 keV of band F and 995 keV of band A.

**Figure 5.23:** New Bands E and F introduced by the DePaul Analysis.
New Bands (G, H, and I)

Double gates around the lower energy levels of band B reveal additional gammas which are not part of band B, but rather branches off of band B which we will call bands G, H, and I. By double gating around the 1140 keV gamma in band G and the 1840 keV crossover gamma between the two bands (see Figure 5.24), we identify the lower energy gammas from band B (178, 654, 735, 884 keV) and the higher energy gammas from band G (1144, 1173, and 1457 keV), and can establish band G.

Figure 5.24: Double gating around 1140 keV and 1840 keV establishes band G.

Similarly, band H is established by double gating around the 1012 keV gamma in the proposed band H and the 884 keV gamma in band B. (see Figure 5.25 below) From this analysis, we can identify several lower band B gammas (574, 504, 178 keV) and several higher level gammas which are proposed to be part of band H (1348 keV and 1675 keV).
Band H decays into a 5306 keV energy level that is also present in band G. From this level, three crossover gammas decay into band B (770, 1397, and 1841 keV). We will verify their existence by double gating between 1140 keV (band G) and 884 keV (band B) in Figure 5.26. In this analysis, smaller peaks exist at 770, 1397, and 1841 keV and this verifies the presence of the crossover gamma rays.

The crossover gamma rays between the lowest energy level in band G (1249 keV and 1840 keV) can be found by using similar double-gating procedures.
The final feature of the DePaul analysis is the establishment of a small branch called band I which decays into the 1990 keV level of band B. It is also important to note that there are some states which decay from the lowest energy levels of band G (5858 keV) and band H (5306 keV) into band I, and eventually into band B. Band I consists of 4 gammas (1950, 1184, 766, and 1650 keV) and the 1950 keV channel competes as a decay channel with the combination of the 1184 and 766 keV channels. A double gate around the 1184 and 766 keV channels of the proposed band I reveals many of the lower level B gammas. Similarly, a double gate around 1950 keV (band I) and 884 keV (band B) reveals many lower level peaks in band B and higher energy peaks in band H, in particular the 1132 keV and 482 keV gammas which serve as a bridge between band H and band I. The final level scheme for band G, H, and I is illustrated in figure 5.27 below.

**Figure 5.27:** Final Level Scheme for new Bands G, H, and I and their link to band B.
The newly confirmed DePaul level scheme, which builds off the original work of Heese and Plettner, is shown below in Figure 5.28.

**Figure 5.28**: Level scheme for $^{73}$Br, as confirmed in the current analysis.
5.2 Relative Efficiency Calibration

In order to properly determine the DCO ratios, we need to account for the variability in detector efficiency as a function of energy for the two groups of detectors used in our analysis. To alleviate this problem, a detector calibration was made using radioactive sources with known energies and intensities, specifically Tantalum-182, Europium-152, and Americium-243. The calibration will determine the detector efficiency for the two groups of detectors used in the DCO analysis as a function of gamma-ray energy. Once these relative efficiencies of the detectors at various angles were established, the measured peak area for a gamma ray observed in the data for $^{73}$Br could be corrected for these efficiency variations. The relative intensity of a particular gamma ray is given by:

$$\text{Intensity} = \frac{\text{PeakArea}}{\text{Efficiency}}$$

Equation 5.2

where the efficiency in equation 5.2 is the efficiency at the energy of the particular gamma-ray of interest. Generally, it was shown that detector efficiency was lower at higher energies, but at very small gamma ray energies, there was also a drop in detector efficiency below about 100 keV. This is shown in the calibration curves included in Appendix A for the side (near 90 degrees) and forward/backward detectors (near 0 and 180 degrees). Using the effit program (Radware), a mathematical model relating detector efficiency (called Total Efficiency) and detector energy(EG) was established with 7 different parameters A through G. The mathematical model is defined by the following set of equations:

$$x = \log(\text{EG}/100)$$

Equation 5.3

$$y = \log(\text{EG}/1)$$

Equation 5.4

$$\text{Efficiency}_1 = (A+B*x)^(-G)$$

Equation 5.5

$$\text{Efficiency}_2 = (D+E*y+F*y^2)^(-G)$$

Equation 5.6

$$\text{Total Efficiency} = 10^{\left([\text{efficiency}_1+\text{efficiency}_2]^(-1/G))\right]}$$

Equation 5.7
The parameters obtained in this calibration are shown below:

**Table 5.1:** Parameter A through G for the Mathematical Model Relating Detector Efficiency with Gamma Ray Energy.

<table>
<thead>
<tr>
<th></th>
<th>Forward Parameters (0, 180°)</th>
<th>Side Parameters (90°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>19.4725</td>
<td>26.8524</td>
</tr>
<tr>
<td>B</td>
<td>12.618</td>
<td>17.1789</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>3.5942</td>
<td>4.548</td>
</tr>
<tr>
<td>E</td>
<td>-0.7715</td>
<td>-0.78</td>
</tr>
<tr>
<td>F</td>
<td>0.078</td>
<td>0.063017</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The calibration curves generated to obtain these parameters with best fit are shown in Appendix A.
5.3 Obtaining DCO Ratios and Interpreting the Results

The second major part of this project uses Directional Correlations of gamma rays from Oriented states of nuclei (DCO) analysis to examine the nature of many of the gamma-ray transitions in the proposed level scheme. In this analysis we will use the definition of the DCO ratio from Equation 4.2 where:

$$\text{DCO} = \frac{I_1}{I_2} = \frac{\text{Intensity of Gamma 1 (0°, 180°) gated by Gamma 2 (90°)}}{\text{Intensity of Gamma 1 (90°) gated by Gamma 2 (0°, 180°)}}$$  \hspace{1cm} \text{Equation 4.2}

Special care must be taken to always assign $\gamma_1$ as the transition between higher energy states than $\gamma_2$. For example, given two gamma transitions from Band A, 583 keV and 804 keV, the 804 keV transition is $\gamma_1$ because the transition occurs between higher energy levels (from 1861 keV to 1057 keV) than the 583 keV transition (from 1057 to 474 keV).

Analysis begins by identifying transitions (or cascades) of gamma rays for which the angular momentum and parity of the states, and multipole character of the gamma rays are known from our energy level diagram for $^{73}\text{Br}$. These sets of transitions will constitute our DCO “calibration data,” which can be organized (or divided) into $L=2 \rightarrow L=2$ cascades and $L=2 \rightarrow L=1$ cascades. It is very important to note that any intermediate transitions between $\gamma_1$ and $\gamma_2$ should be “stretched” $L=2$ transitions which have a theoretical DCO ratio of 1. In general, a stretched transition is one that occurs between states such that $J(\text{initial}) - J(\text{final}) = L$, where $L$ is the multipolarity of the transition. Figure 5.32 shown below illustrates a section of band A which consists of a series of $L=2 \rightarrow L=2$ stretched transitions. It’s important to recognize that an $L=1$ transition can also be stretched, although in this case the relationship would be, $J(\text{initial}) - J(\text{final}) = 1$. 

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As an example, the five curves in Figure 5.30 show the theoretical DCO ratios (derived from electronic communication with Dr. Fischer) of specific gamma ray cascades in which at least one of the two transitions is a stretched $L = 2$ transition. The cascades always have $\gamma_1$ as the transition between higher energy states than the transition $\gamma_2$. The legend identifies the multipolarities for $\gamma_1$ and then $\gamma_2$, so that a label of “L=2, L=1” indicates that $\gamma_1$ is a stretched $L = 2$ transition, and $\gamma_2$ is a stretched $L = 1$ transition. The two curves shown as combinations of dots and dashes each have $\gamma_1$ as a stretched $L = 2$ transition, and $\gamma_2$ as a transition of mixed $L = 1$ and $L = 2$ multipolarities. The so-called mixing ratio indicates the relative amounts of the two types of multipolarities. It is determined from the quantum mechanical matrix elements for the transitions, and can be positive or negative in sign (see Krane text and paper). In particular, the data for the $L = 2 \rightarrow L=2$ cascade are for a $13/2 \rightarrow 9/2 \rightarrow 5/2$ sequence (where the so-called angular alignment
parameter $\sigma$, which quantifies how well on average the angular momentum vector of the initial state is aligned perpendicular to the direction of the incident beam, is given as $\sigma/J = 0.35$, with $J$ as the angular momentum of the initial state). As will be true in all cases, the DCO ratio for this type of stretched $L = 2 \rightarrow L=2$ cascade is unity (or 1) and is independent of the angle $\theta$. This is not true for the other types of cascades, where the value of the DCO ratio depends on $\theta$.

**Figure 5.30**: Theoretical DCO ratios calculated for a simple geometry that places the detectors in a single plane. The target position (or location of the reaction) defines the origin, and $0^\circ$ is defined by the line along which an undeflected beam particle would exit the target. The theoretical DCO ratios in this figure are defined as DCO Ratio $(\gamma_1, \gamma_2) = (\gamma_1 \text{ at } \theta, \gamma_2 \text{ at } \pi/2) / (\gamma_1 \text{ at } \pi/2, \gamma_2 \text{ at } \theta)$, where the $\gamma$-ray cascade is $\gamma_1 \rightarrow \gamma_2$. See main text for details.

The goal of the DCO ratio analysis is to identify the type, or multipole character, of each cascade, and to do so, we would like the values for the ratios of the different types of sequence to be different enough in value so that the sequence type can be determined from the associated experimentally measured DCO ratio. For the simple geometry used to generate the figure, the best choice would be to define the ratio as $(\gamma_1 \text{ at } \theta, \gamma_2 \text{ at } \pi/2) / (\gamma_1 \text{ at } \pi/2, \gamma_2 \text{ at } \theta)$ where $\theta = 0^\circ$. With a large detector array like
Gammasphere, two groups of detectors are selected, with each group covering a range of angles. To maximize the statistics, it is best to include a large number of detectors. This usually means including detectors that are placed at some range of angles near 0 or 180 degrees, or near 90 degrees, as has been done for this analysis. However, as seen from the figure, the greater the range of angles of detectors included, the smaller the difference in values of DCO ratios for the different types of cascades. In other words, there is always an experimental trade-off in selecting how to group the data from detectors in an array.

Once, we have identified known cascades of two transitions which fit the description \( L=2 \rightarrow L=2 \) cascade and \( L=2 \rightarrow L=1 \) cascade, where all transitions are stretched, we can find average values and standard deviation values for the two types of DCO ratios obtained from our GSFMA 123 data set. The results are shown below, both in tabular and graphical formats:

It’s important to note that if the transition is “pure” \( L=2 \), then nearly all gamma ray transitions in nuclei that are \( L=2 \) are electric quadrupole transitions. This implies that there is no change in parity between the initial and final states.

It’s also important to discuss the origin of the uncertainty for these experimentally determined DCO ratios. Uncertainties in DCO Ratios were determined initially using peak area uncertainties from “number of counts” using a Gaussian distribution. The uncertainties represent the square root of the number of counts in a peak (after removing background noise). There was always an error spectrum to align with the background-subtracted gamma spectrum. As a result, the uncertainties arose from the square root of the error spectrum rather than subtracted one.
Table 5.2: DCO Ratios for Known L=2 to L=2 Gamma-Gamma Cascades. The cascade is defined as Gamma 1 → Gamma 2, where Gamma 1 is the transition between the higher energy states. The “Dummy Index” is defined for graphing purposes to differentiate between the DCO ratios for two different cascades that share the same gamma 2 transition.

<table>
<thead>
<tr>
<th>Dummy Index</th>
<th>DCO Ratio</th>
<th>Uncertainty</th>
<th>Gamma 1 Energy (keV)</th>
<th>Gamma 2 Energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>188</td>
<td>1.29</td>
<td>0.04</td>
<td>583</td>
<td>188</td>
</tr>
<tr>
<td>573</td>
<td>1.09</td>
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<td>804</td>
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<td>578</td>
<td>1.17</td>
<td>0.06</td>
<td>995</td>
<td>583</td>
</tr>
<tr>
<td>583</td>
<td>1.19</td>
<td>0.10</td>
<td>1166</td>
<td>583</td>
</tr>
<tr>
<td>588</td>
<td>1.40</td>
<td>0.22</td>
<td>1316</td>
<td>583</td>
</tr>
<tr>
<td>804</td>
<td>0.89</td>
<td>0.01</td>
<td>1166</td>
<td>804</td>
</tr>
<tr>
<td>995</td>
<td>0.97</td>
<td>0.01</td>
<td>1166</td>
<td>995</td>
</tr>
<tr>
<td>1166</td>
<td>1.08</td>
<td>0.03</td>
<td>1316</td>
<td>1166</td>
</tr>
<tr>
<td>503</td>
<td>1.17</td>
<td>0.07</td>
<td>574</td>
<td>503</td>
</tr>
<tr>
<td>570</td>
<td>1.10</td>
<td>0.06</td>
<td>735</td>
<td>574</td>
</tr>
<tr>
<td>578</td>
<td>1.15</td>
<td>0.06</td>
<td>884</td>
<td>574</td>
</tr>
<tr>
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<td>1.15</td>
<td>0.08</td>
<td>884</td>
<td>735</td>
</tr>
<tr>
<td>850</td>
<td>0.97</td>
<td>0.10</td>
<td>953</td>
<td>850</td>
</tr>
<tr>
<td>719</td>
<td>1.10</td>
<td>0.10</td>
<td>953</td>
<td>719</td>
</tr>
<tr>
<td>916</td>
<td>1.11</td>
<td>0.14</td>
<td>953</td>
<td>916</td>
</tr>
<tr>
<td>948</td>
<td>1.08</td>
<td>0.15</td>
<td>1072</td>
<td>953</td>
</tr>
<tr>
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<td>0.86</td>
<td>0.16</td>
<td>1216</td>
<td>953</td>
</tr>
<tr>
<td>452</td>
<td>0.96</td>
<td>0.14</td>
<td>719</td>
<td>462</td>
</tr>
<tr>
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<td>1.19</td>
<td>0.22</td>
<td>850</td>
<td>462</td>
</tr>
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<td>462</td>
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</tr>
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<td>466</td>
<td>0.96</td>
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<td>1071</td>
<td>462</td>
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<tr>
<td>470</td>
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<td>1216</td>
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<td>474</td>
<td>1.00</td>
<td>0.51</td>
<td>1348</td>
<td>462</td>
</tr>
</tbody>
</table>

Average: 1.10 0.14
Standard Deviation: 0.13
Table 5.3: DCO Ratios for Known L=2 to L=1 Gamma-Gamma Cascades. The notation is identical to that used in Table 5.2.

<table>
<thead>
<tr>
<th>Dummy Index</th>
<th>DCO Ratio</th>
<th>Uncertainty</th>
<th>Gamma 1 Energy (keV)</th>
<th>Gamma 2 Energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.89</td>
<td>0.39</td>
<td>583</td>
<td>108</td>
</tr>
<tr>
<td>116</td>
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<td>0.19</td>
<td>188</td>
<td>108</td>
</tr>
<tr>
<td>170</td>
<td>1.45</td>
<td>0.10</td>
<td>574</td>
<td>178</td>
</tr>
<tr>
<td>186</td>
<td>1.46</td>
<td>0.14</td>
<td>735</td>
<td>178</td>
</tr>
<tr>
<td>295</td>
<td>1.61</td>
<td>0.35</td>
<td>462</td>
<td>303</td>
</tr>
<tr>
<td>654</td>
<td>1.38</td>
<td>0.13</td>
<td>574</td>
<td>654</td>
</tr>
<tr>
<td>151</td>
<td>1.49</td>
<td>0.42</td>
<td>574</td>
<td>151</td>
</tr>
<tr>
<td>311</td>
<td>1.49</td>
<td>0.66</td>
<td>953</td>
<td>303</td>
</tr>
</tbody>
</table>

Average: 1.49 0.30
Standard Deviation: 0.21

These results and the associated error bars are depicted in the graph below with known L=2 → L=2 sequences, or cascades illustrated in blue and L=2 → L=1 sequences, or cascades illustrated in red. The value of the dummy index is plotted as the energy of γ2 (horizontal axis) in the figure in order to visually separate DCO ratio data points that share the same transition as γ2 of the cascade. The dummy index does not impact the calculations.
Figure 5.31: DCO Ratio vs. Energy of Gamma 2 Transitions for gamma-gamma cascades of known multipolarities. The data in red (squares) are for known \( L=2 \rightarrow L=1 \) cascades, and the data in blue (circles) are for known \( L=2 \rightarrow L=2 \) cascades. The solid lines show the average values of the measured DCO ratios for the two types of cascades, and the dashed lines indicate one standard deviation above and below the mean.

The results, shown in Figure 5.31, appear relatively close to the theoretical DCO ratios we observed for the simple geometry with the average DCO Ratio for the \( L=2 \rightarrow L=2 \) transitions being 1.10 (14) and the average DCO Ratios for the \( L=2 \rightarrow L=1 \) transitions being 1.49(30). A weighted average was also calculated which accounts for the relative error of each peak obtained in the data collection process.

Our next objective was to apply the information we have gained from the DCO ratios for cascades with known multipolarities to learn something about cascades involving transitions of unknown multipolarity in Bands C, D, E, and F. As was done for the previous cascades, we will require that one of the two transitions in the cascade is known to be a stretched \( L=2 \) transition. We will then measure the DCO
ratio involving transitions with one known (L=2) and one unknown multipolarity, and compare the measured DCO ratio with the ranges of values of DCO ratios for known L=2 → L=2 cascades, and known L=2 → L=1 cascades. Table 5.4 shows the measured DCO ratios involving transitions of unknown multipolarity.
<table>
<thead>
<tr>
<th>Index</th>
<th>Band(s)</th>
<th>DCO Ratio</th>
<th>Uncertainty</th>
<th>Energy (keV)</th>
<th>Multipolarity</th>
<th>Energy (keV)</th>
<th>Multipolarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>C</td>
<td>1.61</td>
<td>0.35</td>
<td>462</td>
<td>2</td>
<td>303</td>
<td>(1)</td>
</tr>
<tr>
<td>308</td>
<td>C</td>
<td>1.49</td>
<td>0.66</td>
<td>953</td>
<td>2</td>
<td>303</td>
<td>(1)</td>
</tr>
<tr>
<td>1204</td>
<td>D</td>
<td>1.19</td>
<td>0.31</td>
<td>1209</td>
<td>?</td>
<td>583</td>
<td>2</td>
</tr>
<tr>
<td>1214</td>
<td>D</td>
<td>1.06</td>
<td>0.04</td>
<td>1209</td>
<td>?</td>
<td>188</td>
<td>2</td>
</tr>
<tr>
<td>1571</td>
<td>D</td>
<td>1.30</td>
<td>0.27</td>
<td>1576</td>
<td>?</td>
<td>583</td>
<td>2</td>
</tr>
<tr>
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<td>D</td>
<td>1.63</td>
<td>0.34</td>
<td>1576</td>
<td>?</td>
<td>188</td>
<td>2</td>
</tr>
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<td>D</td>
<td>1.66</td>
<td>0.32</td>
<td>1618</td>
<td>2</td>
<td>1166</td>
<td>?</td>
</tr>
<tr>
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<td>1368</td>
<td>?</td>
<td>583</td>
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</tr>
<tr>
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<td>0.14</td>
<td>1107</td>
<td>?</td>
<td>976</td>
<td>2</td>
</tr>
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<td>F</td>
<td>1.30</td>
<td>0.33</td>
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</tr>
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<td>1050</td>
<td>?</td>
</tr>
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<td>F</td>
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<td>0.24</td>
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<td>?</td>
</tr>
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<td>0.23</td>
<td>846</td>
<td>?</td>
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<td>2</td>
</tr>
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<td>846</td>
<td>E</td>
<td>1.08</td>
<td>0.31</td>
<td>846</td>
<td>?</td>
<td>188</td>
<td>2</td>
</tr>
<tr>
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<td>E</td>
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<td>0.41</td>
<td>819</td>
<td>?</td>
<td>904</td>
<td>?</td>
</tr>
<tr>
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<td>E</td>
<td>1.33</td>
<td>0.25</td>
<td>960</td>
<td>?</td>
<td>904</td>
<td>?</td>
</tr>
<tr>
<td>819</td>
<td>E</td>
<td>1.00</td>
<td>0.49</td>
<td>819</td>
<td>?</td>
<td>525</td>
<td>?</td>
</tr>
<tr>
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<td>E</td>
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<td>0.54</td>
<td>960</td>
<td>?</td>
<td>1665</td>
<td>?</td>
</tr>
<tr>
<td>E</td>
<td>1.43</td>
<td>0.22</td>
<td>820</td>
<td>?</td>
<td>583</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>1.52</td>
<td>0.25</td>
<td>820</td>
<td>?</td>
<td>188</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>1.27</td>
<td>0.39</td>
<td>820</td>
<td>?</td>
<td>525</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>1.06</td>
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<td>1.47</td>
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<td>1112</td>
<td>?</td>
<td>188</td>
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<td></td>
</tr>
<tr>
<td>E, F</td>
<td>1.86</td>
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<td>525</td>
<td>?</td>
<td>188</td>
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<td>E, F</td>
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<td>0.70</td>
<td>976</td>
<td>?</td>
<td>525</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>?</td>
<td>1.35</td>
<td>0.16</td>
<td>592</td>
<td>?</td>
<td>188</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>F, E?</td>
<td>1.77</td>
<td>0.12</td>
<td>901, 904?</td>
<td>?</td>
<td>188</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>E, F</td>
<td>1.44</td>
<td>0.23</td>
<td>901, 904?</td>
<td>?</td>
<td>525</td>
<td>?</td>
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</tr>
<tr>
<td>?</td>
<td>1.80</td>
<td>0.31</td>
<td>712</td>
<td>?</td>
<td>144</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>?</td>
<td>1.62</td>
<td>0.28</td>
<td>712</td>
<td>?</td>
<td>516</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

*Table 5.4: DCO Ratios Involving A Transition of Unknown Multipolarity*
We begin by examining the 1209 keV transition between Band D and Band A, which is shown in figure 5.32 above. The DCO ratio for the 1209 and 583 keV transitions was measured to be 1.19 +/- 0.31. Note that this cascade really consists of a 1209 → 995 → 804 → 583 keV transitions, but since each of the three transitions within band A are of multipolarity L=2, and because the DCO ratio for L=2 → L=2 cascades is 1, it is valid to use the measured value of the DCO ratio for the 1209-583 keV pair to determine the multipolarity of the 1209 keV transition. The two measured DCO ratios involving the 1209 keV, stated in Table 5.4 and shown in Figure 5.33 at energies of approximately 1209 keV, indicate that these are L=2 → L=2 cascades.
Next consider the 1575 keV transition in Band D of the same figure. As indicated in Table 5.4, the 1575 keV transition is considered to be gamma 1 and the lower energy transitions (gamma 2) selected for the DCO ratios are L = 2 transitions. As a result, we cannot use the interval for L=2 → L=1 cascades, but we know that the L=1 → L=2 cascades will have the reciprocal relationship, so let’s add this range and type of cascade to our previous chart. Our measured results for the DCO ratios of the 1575 keV transition are 1.30 (27) and 1.63 (34) as shown in Figure 5.33 at energies slightly below and above 1575 keV, and while these lie just outside our range of one standard deviation from the DCO ratios for L=2 → L=2 cascades, they are far outside the allowed range for an L=1 → L=2 cascade. While a bit problematic, we tentatively assign the 1575 keV transition (L=2) multipolarity.

**Figure 5.33:** DCO Ratio vs. Energy (using the dummy index of Table 5.4) for DCO ratios involving the 303, 1209, 1575, and 1618 keV transitions of unknown multipolarity. See Table 5.4 for details. DCO ratios in red indicate the range for L=2 → L=1 cascades; in blue indicate the range for L=2 → L=2 cascades; in green indicate the range for L=1 → L=2 cascades.
Next consider the known L=2 1618 keV transition from Band D to Band A. In this case, γ1 is the 1618 keV transition, the γ2 is the 1166 transition, with unknown multipolarity, and the experimentally obtained DCO ratio is 1.66+/−0.32, as shown in Figure 5.33. The L=2 → L=1 option is not a possibility because the lower transition is a known L=2 transition. This is outside of our expected range for L=2 → L=2 transitions, but clearly eliminates the L=1 → L=2 option which ranges from 0.46 to 0.88. Although this is not a perfect fit, it seems that the data favors the assignment of L=2 → L=2 to this transition and an assignment of L=2 to the 1209, 1575, and 1618 keV transitions. As a result, we suggest that the previously proposed spin assignments for this band have been verified by our own DCO analysis. We can remove the parentheses of the tentative assignment (29/2+) for energy level 5640 in Band D and confirm the angular momentum assignment.

Next, we will consider the 303 keV transition, as shown in Figure 5.34 below.

![Figure 5.34: Lower Energy Transitions in 73Br Level Scheme.]

There are two DCO ratios that include the 303 keV transition as γ2 with a stretched quadrupole transition as γ1. The direct transition from a γ1 of 462 keV has a DCO ratio of 1.61 +/- 0.35. The other transition, with γ1 of 953 keV has a string of quadrupoles between itself and the 303 keV transition. Its DCO ratio is 1.49 +/- 0.66. The DCO ratios are indicated in Figure 5.33. Both cascades are consistent with the L=2 → L=1 assignment. This result confirms the previous tentative assignment of (L=1) for the 303 keV transition.

Next, consider the DCO results in band F. The discussed elements of band F are shown in figure 5.35.
Figure 5.35: Lower Energy Transitions between Band F and Band A.

Let’s consider the DCO ratio for the cascade including the 1368 keV (unknown L) and 583 keV (L=2) transitions, which has a value of 0.59 +/- 0.20. This result favors the assignment of L=1 \rightarrow L=2. Note that for this cascade, $\gamma_2$ is the known L=2 transition. The DCO ratio for the sequence including the 1107 keV (unknown L) and 976 keV (L=2) transitions is 0.53 +/- 0.14 which would also suggest a L=1 \rightarrow L=2 assignment. This result does not agree with the observation that the 1107 and 976 keV transitions seem to be part of a regular rotational structure, and so are expected to each be stretched E2 in character. The DCO ratio for these transitions is a bit puzzling and deserves further attention. Future work with angular distributions of single gamma rays, which typically are higher in statistics, can be done in an attempt to clarify the inconsistencies in these results.

In order to identify the multipolarity of the 1050 keV transition of band F, we will assume (based on the apparent rotational structure of the band) that the 901 keV, 976 keV, and the 1107 keV represent a stretched E2 cascade, and each of these transitions are L=2. Given that assumption, we examine the DCO Ratio for the 976-1050 sequence, which is 0.72 +/- 0.34. This result favors the assignment of L=2 \rightarrow L=1, with expected DCO ratios in the range of 0.46 to 0.88. As a result, we can assign the 1050 keV transition as L=1. Based on the ratios we have identified and the expected E2 stretched structure of the band, we can make tentative assignments.
of $(11/2)$ to the $1524$ keV energy level and $(15/2)$ to the level at $2425$ keV. These are most likely positive parity, but we cannot assign the parity with certainty.

We will now consider band E, which is shown below in Figure 5.36 and possesses a number of inherent analytical difficulties. The transitions between band E and band A, and all of those below the state at $1903$ keV in band E, may not be stretched, so we can only trust the DCO ratios of direct sequences without any intermediate unknown transitions. Also, there are two $525$ keV transitions and most DCO ratios involving a $525$ keV gamma will include the contributions of both of these gammas. We do not have multipolarity information on the $713$ keV gamma, but we will attempt to give a range of possible spin assignments for the lower levels in Band F and try to determine the most likely angular momentum values for a variety of related states.

*Figure 5.36: Lower Energy Transitions between Band E and Band F.*
We make two assumptions as we proceed in establishing the angular momentum values for different states. Our first assumption is that the angular momentum values do not increase or decrease by more than two units of angular momentum between the initial and final states. Transitions involving larger changes (or differences) in angular momentum between excited states in nuclei are relatively rare, and are associated with isomeric states that have half-lives much longer than would be observed in this data set. This is a reasonable assumption because these levels are not isomeric and have half-lives that are typically associated with $L = 1$ and $L = 2$ transitions. The second assumption is that the angular momentum values will generally increase as the energy level of the state increases, as is typical for states populated through the type of compound nuclear reaction used in this study to create the $^{73}$Br. Recall that angular momentum values cannot be directly measured, but are deduced by studying the multipolarity of the transition from an initial state to a final state.

Begin the analysis of band E by considering the 999 keV state which is fed from above by the transition from the 1524 keV state with angular momentum $(11/2^+)$ and decays to states of known angular momentum value $5/2^+$ and $9/2^+$. Assuming all $L=1$ or $L=2$ transitions, we can state that the 999 keV energy level has two possible angular momentum values: $(7/2)$, or $(9/2)$. Moving up in energy level, we next consider the possible angular momentum values for the 1903 keV energy level. If the 904 keV transition from 1903 keV to 999 keV is either $L = 1$ or $L = 2$, then the 1903 keV state is either $(13/2)$, $(11/2)$, or $(9/2)$. If we subsequently determine that the 999 keV state is $(7/2)$, it becomes highly unlikely that the 1903 keV state is $(13/2)$ because this would require an $L=3$ transition. Next, we consider the 846 keV gamma which decays from the 1903 keV state to 1057 keV state in Band F which has a known angular momentum of $13/2^+$. Since it is unlikely, though still possible, that $\Delta L = 0$, we can hypothesize that the 1903 keV is most likely not 13/2, and as a logical consequence, the 999 keV state is not 7/2. So we will begin with the
hypothesis that the 1903 keV state is (11/2) or (9/2) and the 999 keV state is (9/2) and study the DCO ratios to see if they support or disprove this claim.

The next step is to examine the 2722 keV energy level and the 819 keV and 1665 keV transitions that stem from it. Since the 819 keV lies as part of what appears to be a rotational band, the 819 keV should be L = 2. Next we examine the DCO ratio for the gamma pair 819 keV and 904 keV and find it to be 0.84 +/- 0.41 which is consistent with an L=2 \rightarrow L=2 sequence. We also consider the gamma pair 960 keV and 904 keV and find its ratio to be 1.33 +/- 0.25 which is also consistent with an L=2 \rightarrow L=2 cascade.

Let’s also consider the gamma pair with 960 and 1665 keV, which has a ratio of 0.67 +/- 0.54. We have conjectured that the 960 keV (\gamma_1) is L=2, so while this is a very small ratio for an L=2 \rightarrow L=2 sequence, the fairly large error bars do allow for an L = 2 assignment for the 1665 keV transition and eliminate calling this an L=2 \rightarrow L=1 transition. As a result, we can assign an angular momentum of (17/2) to the 2722 keV state. This also seems to be consistent with our previous argument for the (13/2) assignment to the 1903 keV state and the (9/2) for the 999 keV state. We can also infer positive parity because of the stretched L=2 nature of the 1665 keV transition to the known 13/2+ state.

We should also consider DCO ratios involving the 846 keV transition as \gamma_2 in the ratio. The DCO ratio that takes \gamma_1 as the 583 keV transition is 0.94 +/- 0.23. In a second case, the \gamma_1 is 188 keV (but it passes through the L=2 583 keV transition) and has the DCO ratio 1.08 +/- 0.31. Both of these results would point to an L=2 \rightarrow L=2 cascade, but this does not appear to agree with our proposed assignments of (13/2+) \rightarrow 13/2+. One possible explanation that does not rule out our proposed hypothesis is that the 846 keV transition may not be a single multipolarity, but rather that there may be mixing of two multipolarities (e.g., electric quadrupole and magnetic dipole mixing is very common) which can also result in DCO ratios near 1.
Future research should examine the possibility of analyzing gamma-ray angular distribution data to determine whether this transition might have a mixed multipolarity.

The final DCO ratio to consider in this band is the one for the 819 keV and 525 keV transitions. There is an intermediate stretched transition 904 keV gamma that should not affect the DCO ratio since it is $L = 2$. The DCO ratio is $1.00 \pm 0.49$. In this scenario, the 525 keV is not stretched $L = 2$ because we have shown there to be a $13/2^+ \rightarrow 9/2^+ \rightarrow 9/2^+$ cascade. Therefore, we can conclude that the measured ratio is indicating either $L = 1$ multipolarity for the 525 keV transition, which is allowed due to the large uncertainty, or a mixed $L = 1/L = 2$ multipolarity.

In conclusion, we have examined the DCO ratios for known transitions and established the following guidelines for identifying transitions.

**Table 5.5: Empirically Defined DCO Ratios.**

<table>
<thead>
<tr>
<th>Transition</th>
<th>Average</th>
<th>+/- Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L=2 \rightarrow L=1$</td>
<td>1.49</td>
<td>0.21</td>
</tr>
<tr>
<td>$L=2 \rightarrow L=2$</td>
<td>1.10</td>
<td>0.13</td>
</tr>
<tr>
<td>$L=1 \rightarrow L=2$</td>
<td>0.67</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Using these guidelines and additional interpretation techniques, we have tentatively identified (parentheses) or confirmed the following angular momentum assignments.

**Table 5.6: Angular Momentum Assignments from this Analysis.**

<table>
<thead>
<tr>
<th>Energy of State (keV)</th>
<th>Band</th>
<th>Angular Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>999</td>
<td>E</td>
<td>9/2+</td>
</tr>
<tr>
<td>1903</td>
<td>E</td>
<td>13/2+</td>
</tr>
<tr>
<td>2722</td>
<td>E</td>
<td>17/2+</td>
</tr>
<tr>
<td>3682</td>
<td>E</td>
<td>21/2+</td>
</tr>
<tr>
<td>4794</td>
<td>E</td>
<td>(25/2+) based on rotational band structure</td>
</tr>
<tr>
<td>6044</td>
<td>E</td>
<td>(29/2+)</td>
</tr>
<tr>
<td>7539</td>
<td>E</td>
<td>(33/2+)</td>
</tr>
<tr>
<td>9280</td>
<td>E</td>
<td>(37/2+)</td>
</tr>
<tr>
<td>1524</td>
<td>F</td>
<td>11/2</td>
</tr>
<tr>
<td>2425</td>
<td>F</td>
<td>15/2</td>
</tr>
<tr>
<td>3401</td>
<td>F</td>
<td>19/2</td>
</tr>
<tr>
<td>4508</td>
<td>F</td>
<td>(23/2*) based on rotational band structure</td>
</tr>
<tr>
<td>5772</td>
<td>F</td>
<td>(27/2)</td>
</tr>
<tr>
<td>7147</td>
<td>F</td>
<td>(31/2)</td>
</tr>
<tr>
<td>178</td>
<td>---</td>
<td>3/2-**change from tentative to confirmed</td>
</tr>
<tr>
<td>5640</td>
<td>D</td>
<td>29/2+*change from tentative to confirmed</td>
</tr>
</tbody>
</table>

The final level scheme with the newly confirmed and tentative assignments is shown in Figure 5.37. Additional work will be done in future research to identify
additional elements of the level scheme and verify any tentative angular momentum assignments made in this project.

**Figure 5.37**: Finalized Level Scheme for $^{73}$Br which includes new bands and new angular momentum assignments.
Chapter 6: Data Interpretation of the Rotational Bands

Classically, the moment of inertia of a body indicates its ability to resist changes to its angular momentum around an axis of rotation. Imagine a person sitting on a spinning stool in the physics lab holding a set of small weights. If their hands are outstretched, then the mass from the weights is distributed further from the axis of rotation than if they hold the masses close to their body. It is more difficult to rotationally accelerate the person with the outstretched hands, so this person’s moment of inertia will be higher. In Chapter 2, we showed that nuclei can often exhibit the properties of a quantum mechanical rigid rotor. The nucleus can be characterized by a moment of inertia, and the set of excited states in a rotational band represents the rotation of the nucleus about some axis at increasingly larger angular speeds or momentum. If we collect data for the excitation energies of these states as a function of their angular momentum, we can extract a value for the moment of inertia of a particular rotational band using equation 2.5 for a quantum mechanical rotor.

It is common to model the nucleus of an atom as a sphere with a moment of inertia of:

\[ I = \frac{2}{5} MR^2 \quad \text{Equation 6.1} \]

Let’s begin with this assumption and then extend this to allow for simple deformed (non-spherical) shapes. We need a way to approximate the average radius of a particular nucleus. Then we can look for deviations from this spherical shape. It is important to note that the nucleus of an atom does not have a precisely defined radius. A similar issue also arises with measurement of an atomic radius, as the radius is often measured with the atom in a compound and notable variation occurs for the measurement of an atomic radius when it occurs in different atoms. As stated in Krane (Introductory Nuclear Physics, Chapter 5), the density of atomic nuclei is relatively constant over short distances from the center of the nucleus and
then drops rapidly to zero. The approach to quantifying the nuclear radius is to
divide it into two parts, the “mean radius”, where the density is half of its central
maximum and the “skin thickness” over which the density drops from near its
maximum to near its minimum. Krane also states that nuclear particles seem to be
evenly distributed throughout the nucleus, maintaining a relatively constant
density. We will define a nuclear density \( k \) as being the number of nuclear
particles per spherical volume.

\[
\frac{A}{\frac{4}{3} \pi R^3} \approx k \quad \text{Equation 6.2}
\]

Since this nuclear density is constant, the number of nuclear particles \( A \) (neutrons +
protons) is proportional to the radius \( R \) cubed. This can also be expressed as:

\[
R = R_o A^{\frac{1}{3}} \quad \text{Equation 6.3}
\]

where \( R_o \) is taken to be a constant often given as 1.2 fm. The value of \( R_o \) is
determined by considering measured values for the nuclear radius over a wide
range of masses (\( A_s \)), and values in the range of 1.1-1.5 fm are reasonable.

Normally a nucleus is modeled as a sphere with a volume of:

\[
\frac{4}{3} \pi R^3 \quad \text{Equation 6.4}
\]

and a moment of inertia that of a solid sphere is:

\[
I_{sphere} = \frac{2}{5} M R^2 = \frac{2}{5} M R_o^2 A^{\frac{2}{3}} \quad \text{Equation 6.5}
\]
For the $^{73}$Br nucleus, the moment of inertia should be calculated with

$$A = 73$$

$$M = 73 \times 931.5 = 67999.5 \text{ MeV/c}^2 \quad \text{Equation 6.6}$$

A nucleus will take on a deformed shape rather than a spherical shape if, in doing so, it can lower its energy. This deformation results in the occurrence of rotational bands and the degree to which it occurs can be measured using the deformation parameter ($\beta$) in the following equation:

$$I(\text{deformed}) = I(\text{spherical}) \times (1 + 0.31\beta) \quad \text{Equation 6.7}$$

The deformations can be modeled as spheroids (ellipsoids of revolution) in which we can assume axial symmetry. This deformation parameter is also related to the eccentricity of the ellipse. In general if $\beta > 0$, the nucleus is a prolate ellipsoid and if $\beta < 0$, it is considered an oblate ellipsoid.

The surface of the shape can be described by using the $Y_{20}$ spherical harmonic in the equation for the radius of the ellipsoid and substituting this into the equation for the moment of inertia of a sphere. A detailed summary of the mathematical origin of the 0.31 in front of the $\beta$ parameter can be found in Appendix C and is taken from Krane.

Our goal is to determine the approximate deformation, in terms of a value of $\beta$, for each of the rotational bands in $^{73}$Br. In Band A, the graph of Energy vs. $J(J+1)$ is shown in Fig. 2.3. We use the slope to extract a value for (deformed) moment of inertia and compare it the value calculated for an idealized rigid sphere, found with equations 6.3 and 6.5, with $R_0 = 1.3$ fm. The deformation parameter $\beta$ for band A was determined to be 0.47.
Fig. 6.1 shows a plot of level energy versus \( J(J + 1) \) for band C, along with a linear fit to the data. While the data do support a linear relationship, the lowest two energy states systematically diverge from linearity. The slope of the energy versus \( J(J + 1) \) data at the bottom of the rotational band is different from the slope at the top of the band, indicating that the moment of inertia of the band changes as the nucleus gains in angular momentum. For this analysis, we will remove the lower two energy levels from the fit, and extract a moment of inertia and deformation parameter for the band above these states.

![C Band Level Energy vs. J(J+1)](image)

**Figure 6.1:** Data from the so-called “C Band” of \(^{73}\text{Br}\). The level energies in keV vs the quantity \( J(J + 1) \), where \( J \) is the total angular momentum of the corresponding level, are plotted. The linear relationship between the level energy and \( J(J + 1) \) supports the characterization of this sequence of energy levels as belonging to a quantum mechanical rigid rotor.

We repeat the same analysis for our other known bands, Bands B and D, and we identify the deformation parameters for each of these and display the results in Table 6.1. (Additional graphs and tables may be found in Appendix B)
**Table 6.1: Approximate Deformation Parameters for bands of Known Parity.**

<table>
<thead>
<tr>
<th>Band</th>
<th>Deformation Parameter Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.47</td>
</tr>
<tr>
<td>B</td>
<td>0.22</td>
</tr>
<tr>
<td>C</td>
<td>0.13</td>
</tr>
<tr>
<td>D</td>
<td>0.49</td>
</tr>
</tbody>
</table>

An interesting observation was noted in this analysis. The bands with a greater measured deformation (bands A and D) are the positive parity bands, while the bands with a smaller measured deformation (bands B and C) are the negative parity bands. This observation about the deformation parameter differences between positive and negative parity bands provides additional evidence that the rotational bands built on positive parity shell model states differ greatly from the rotational bands built on negative parity shell model states.

Using this discovery, we now examine the parity of our unknown bands E and F. It has been suggested from the DCO analysis that the parity of band E is positive, but it is worthwhile to calculate the deformation parameter based on the tentative assignments made in this thesis. Based on the graph of Energy vs. $J(J+1)$ shown below in Figure 6.2 and excluding the data for the lowest energy level of the band, we identify a deformation parameter $\beta$ of 0.40, a relatively large deformation which is consistent with our hypothesis that positive parity bands tend to exhibit greater deformation.
**Figure 6.2:** Data from the so-called "E Band" of $^{73}$Br. The level energies in keV vs the quantity $J(J + 1)$, where $J$ is the total angular momentum of the corresponding level, are plotted. The linear relationship between the level energy and $J(J + 1)$ supports the characterization of this sequence of energy levels as belonging to a quantum mechanical rigid rotor.

Finally, let’s calculate the deformation parameter $\beta$ for band F based on the hypothesized energy levels from our analysis. The graph of Energy vs. $J(J+1)$ is shown below in Figure 6.3 and the deformation parameter $\beta$ (again removing lower energy levels) is determined to be 0.52. This deformation is somewhat larger than calculated in the other bands, but still consistent with the deformations for the other positive parity bands. Extracting approximate values for the deformation parameters of these bands allows us to assign positive parity to the new bands E and F.

Adopting these new parity assignments for bands E and F, the final level scheme for $^{73}$Br as determined from the current analysis is presented in Figure 6.3.
Figure 6.3: Data from the so-called “F Band” of $^{73}$Br. The level energies in keV vs the quantity $J(J + 1)$, where $J$ is the total angular momentum of the corresponding level, are plotted. The linear relationship between the level energy and $J(J + 1)$ supports the characterization of this sequence of energy levels as belonging to a quantum mechanical rigid rotor.
Chapter 7: Conclusion and Next Steps

Several important experimental outcomes resulted from this analysis, beginning with the identification of new bands in the level scheme for $^{73}$Br, including bands E, F, G, H, and I. Band F was especially difficult to establish because of the presence of two 525 keV gamma rays in succession. Several crossover gamma pathways were identified between these newly identified bands and the previously published band A, B, C, and D. The second critical outcome was the use of DCO Ratios to identify the multipolarity of 10 gamma ray emissions, and discern from these, 16 angular momentum and parity assignments for states in bands D, E, and F (see table 5.5). The third and final outcome was the observation of a strong correlation between the magnitude of the deformation parameter $\beta$ and the parity of a chosen band. When the deformation parameter was evaluated, a larger $\beta$ correlated well with a positive band parity and a smaller $\beta$ correlated well with a negative band parity. This information will be helpful as future bands in $^{73}$Br are identified.

The primary experimental challenge faced in this research occurred because of a lack of sufficient statistics for many of the gamma-gamma cascades, particularly in establishing DCO ratios from the GSFMA123 data set. There were also limitations in determining accurate (and/or precise) energies for gamma rays at high energies in the GSFMA19 data set. An expensive approach might involve repeating these experiments with more sensitive gamma ray detectors, or simply collecting data at higher rates or for a longer duration.

Another option to be considered for future analysis is using the angular distributions of the intensity of gamma radiation to identify the multipolarity associated with that transition. A previous DePaul student (see Schmidt thesis) used this analysis to study the $^{73}$Kr nucleus. Based on electromagnetic theory, the intensity of radiation as a function of angle is given by:
$I(\theta) = A_0 + A_2P_2(\cos\theta) + A_4P_4(\cos\theta)$ \hspace{1cm} \text{Equation 7.1}

where $A_0$ is an overall normalization and $A_2$ and $A_4$ contain information critical to identifying multipolarity assignments. This technique could be applied to the GSFMA123 data set or future data sets to better identify the nature of the electromagnetic radiation during transitions in the $^{73}$Br nucleus. An angular distribution analysis does not impose the same kind of strict coincidence requirements that are relied upon in a DCO ratio analysis. This means that for the same data set, an angular distribution analysis for a particular transition will include higher statistics than a DCO analysis involving the same transition that is, by definition, part of a gamma-gamma cascade. It should be noted that this gain in statistics is often overpowered by the complexity of the so-called “singles” (ungated) gamma ray spectra. In other words, the peak for the gamma ray of interest is often difficult to resolve from many other nearby gamma rays. For this reason, DCO ratios are often quite powerful, even with poor statistics. The work in this thesis is an example of the usefulness of this technique.
References


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Paul, Eddie, http://ns.ph.liv.ac.uk/mass110.html


Schmidt, David, “Exploring the shape of the Nucleus$^{73}$Kr,” (2002)

Svelnys, Derrick, Master’s Thesis at DePaul University, “A Spectroscopic Investigation of the Excited States of the Nucleus $^{72}$Se” (2005)
Appendix A: Calibration Detector Efficiency with Gamma Energy

Figures A1 and A2 show that the fit for this calibration of detector efficiency with gamma energy for the forward and side detectors. The mathematical model that aligns with this calibration is located in Chapter 5, section 2.

Figure A1: Calibration Curve for the Forward Detectors showing the relative efficiency of the detectors as a function of gamma-ray energy (in keV).
Figure A2: Calibration Curve for the Side Detectors showing the relative efficiency of the detectors as a function of gamma-ray energy (in keV).
Appendix B: Additional Graphs and Tables for Nuclear Deformation Analysis

A Band

<table>
<thead>
<tr>
<th>Level Energy</th>
<th>Initial Angular Momentum</th>
<th>J(J+1)</th>
<th>Level Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>8460</td>
<td>18.5</td>
<td>360.75</td>
<td>8460</td>
</tr>
<tr>
<td>6806</td>
<td>16.5</td>
<td>288.75</td>
<td>6806</td>
</tr>
<tr>
<td>5338</td>
<td>14.5</td>
<td>224.75</td>
<td>5338</td>
</tr>
<tr>
<td>4022</td>
<td>12.5</td>
<td>168.75</td>
<td>4022</td>
</tr>
<tr>
<td>2856</td>
<td>10.5</td>
<td>120.75</td>
<td>2856</td>
</tr>
<tr>
<td>1861</td>
<td>8.5</td>
<td>80.75</td>
<td>1861</td>
</tr>
<tr>
<td>1057</td>
<td>6.5</td>
<td>48.75</td>
<td>1057</td>
</tr>
</tbody>
</table>

I actual of A Band = 23.695

Ideformed = \(\frac{2}{5MR^2}(1+.31\text{Beta})\)

Beta = 0.468728812

A Band Level Energy vs. J(J+1)

\[ y = 23.695x - 34.866 \]

\[ R^2 = 0.99968 \]
B Band

<table>
<thead>
<tr>
<th>Level Energy</th>
<th>Initial Angular Momentum</th>
<th>J(J+1)</th>
<th>Level Energy</th>
</tr>
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<td>11299</td>
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<td>11299</td>
</tr>
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<td>15.5</td>
<td>255.75</td>
<td>6404</td>
</tr>
<tr>
<td>5091</td>
<td>13.5</td>
<td>195.75</td>
<td>5091</td>
</tr>
<tr>
<td>3909</td>
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<td>143.75</td>
<td>3909</td>
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<td>99.75</td>
<td>2874</td>
</tr>
<tr>
<td>1990</td>
<td>7.5</td>
<td>63.75</td>
<td>1990</td>
</tr>
</tbody>
</table>

I actual of B Band = 22.082

Ideformed = \((2/5MR^2)\times(1+0.31\beta)\)

\(\beta = 0.217229112\)

![B Band Level Energy vs. J(J+1) Graph](image)
C Band

<table>
<thead>
<tr>
<th>Level Energy</th>
<th>Initial Angular Momentum</th>
<th>J(J+1)</th>
<th>Level Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>10161</td>
<td>20.5</td>
<td>440.75</td>
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<tr>
<td>8565</td>
<td>18.5</td>
<td>360.75</td>
<td>8565</td>
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<tr>
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<td>288.75</td>
<td>7100</td>
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<tr>
<td>5752</td>
<td>14.5</td>
<td>224.75</td>
<td>5752</td>
</tr>
<tr>
<td>4536</td>
<td>12.5</td>
<td>168.75</td>
<td>4536</td>
</tr>
<tr>
<td>3465</td>
<td>10.5</td>
<td>120.75</td>
<td>3465</td>
</tr>
<tr>
<td>2512</td>
<td>8.5</td>
<td>80.75</td>
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<tr>
<td>1662</td>
<td>6.5</td>
<td>48.75</td>
<td>1662</td>
</tr>
</tbody>
</table>

I actual of C Band = 21.543

Ideformed = (2/5MR^2)*(1+.31Beta)

Beta = 0.133187985
$y = 21.543x + 799.61$

$R^2 = 0.99861$
D Band (uses hypothesized angular momentum states)

<table>
<thead>
<tr>
<th>Level Energy</th>
<th>Initial Angular Momentum</th>
<th>J(J+1)</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>12677</td>
<td>22.5</td>
<td>528.75</td>
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<tr>
<td>10799</td>
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<tr>
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<td>7251</td>
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<td>14.5</td>
<td>224.75</td>
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<tr>
<td>4065</td>
<td>12.5</td>
<td>168.75</td>
<td>4065</td>
</tr>
</tbody>
</table>

I_{actual} of D Band = 23.836

I_{deformed} = \frac{2}{5MR^2}(1+0.31\beta)

\beta = 0.490713596
<table>
<thead>
<tr>
<th>Level Energy</th>
<th>Initial Ang Momentum</th>
<th>J(J+1)</th>
<th>Level Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>9280</td>
<td>18.5</td>
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<td>3682</td>
</tr>
<tr>
<td>2722</td>
<td>8.5</td>
<td>80.75</td>
<td>2722</td>
</tr>
</tbody>
</table>

I actual of E Band = 23.282

I_{deformed} = \left(\frac{2}{5MR^2}\right) \times (1 + 0.31\ Beta)

Beta = 0.40433662

**E Band Energy Level vs. J(J+1)**

\[ y = 23.282x + 847.82 \]

\[ R^2 = 0.99986 \]
F Band

<table>
<thead>
<tr>
<th>Level Energy</th>
<th>Initial Ang</th>
<th>J(J+1)</th>
<th>Level Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>7147</td>
<td>15.5</td>
<td>255.75</td>
<td>7147</td>
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<tr>
<td>5772</td>
<td>13.5</td>
<td>195.75</td>
<td>5772</td>
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<tr>
<td>4508</td>
<td>11.5</td>
<td>143.75</td>
<td>4508</td>
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<tr>
<td>3401</td>
<td>9.5</td>
<td>99.75</td>
<td>3401</td>
</tr>
</tbody>
</table>

I actual of F Band = 24.008

Ideformed = (2/5MR^2)*(1+.31Beta)

Beta = 0.517531915

\[ y = 24.008x + 1035.6 \]
\[ R^2 = 0.99955 \]
Appendix C: Origin of 0.31 Constant for the Moment of Inertia Equation (Krane)

Assuming the nucleus takes the shape of an ellipsoid of revolution, we can describe the surface of the shape using the $Y_{20}$ spherical harmonic as:

$$ R(\theta, \phi) = R_{\text{avg}} [1 + \beta Y_{20}(\theta, \phi)] $$

where $Y_{20}(\theta, \phi) = \frac{5}{\sqrt{16\pi}} (3\cos^2 \theta - 1)$.

Note that there is no axial dependence in this equation (no dependence on $\phi$).

If we then square the radial term and truncate to the first-order term:

$$ R^2 = R_{\text{avg}}^2 [1 + \beta Y_{20}(\theta, \phi)]^2 = R_{\text{avg}}^2 [1 + 2\beta Y_{20} + \beta^2 Y_{20}^2] $$

we will arrive at:

$$ R^2 = R_{\text{avg}}^2 (1 + 2\beta Y_{20}) $$

Finally, calculate the expectation value or average of $Y_{20}$ over all $\theta$, arriving at:

$$ \langle Y_{20}(\theta, \phi) \rangle = \frac{5}{\sqrt{16\pi}} \left[ (3\cos^2 \theta - 1) \right] = \frac{5}{\sqrt{16\pi}} (3 \times 0.5 - 1) = \frac{5}{16\pi} \frac{1}{2} $$

Next, we substitute this into $R^2 = R_{\text{avg}}^2 (1 + 2\beta Y_{20})$ and it yields:

$$ R^2 = R_{\text{avg}}^2 \left[ 1 + 2\beta \sqrt{\frac{5}{16\pi} \frac{1}{2}} \right] = R_{\text{avg}}^2 \left[ 1 + \beta \sqrt{\frac{5}{16\pi}} \right] = R_{avg}^2 (1 + 0.315\beta) $$

Recalling the aforementioned moment of inertia equation, we can clearly see where the 0.31$\beta$ parameter arises from:

$$ I_{\text{rigid}} = \frac{2}{5} MR^2 = \frac{2}{5} MR_{\text{avg}}^2 (1 + 0.31\beta) $$

Equation 6.7